EMMITANCE MEASUREMENT AND ERROR STUDY

21.08.2026

Ramazan Tunç

WHAT IS A BEAM

A beam is a collection of particles traveling together.
The motion of each particle is described relative to a reference particle.
A single particle can be described by a six-dimensional phase-space vector

$$\mathbf{X} = \begin{pmatrix} x \\ x' \\ y \\ y' \\ z \\ \delta \end{pmatrix}$$

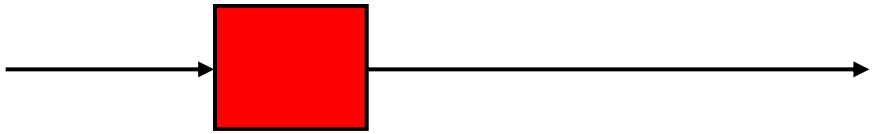
- x, y: transverse positions
- z: longitudinal position
- x', y': transverse trajectory slopes
- δ: momentum deviation

$$x' = \frac{dx}{dz} = \frac{p_x}{p_z} = \tan\theta_x, \qquad y' = \frac{dy}{dz} = \frac{p_y}{p_z} = \tan\theta_y$$

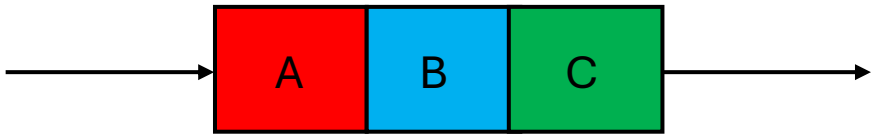
$$\delta = \frac{p - p_0}{p_0}$$

Beam Transport

A particle's coordinates change as it moves through the beamline.



$$\mathbf{X}_{out} = R\mathbf{X}_{in}$$

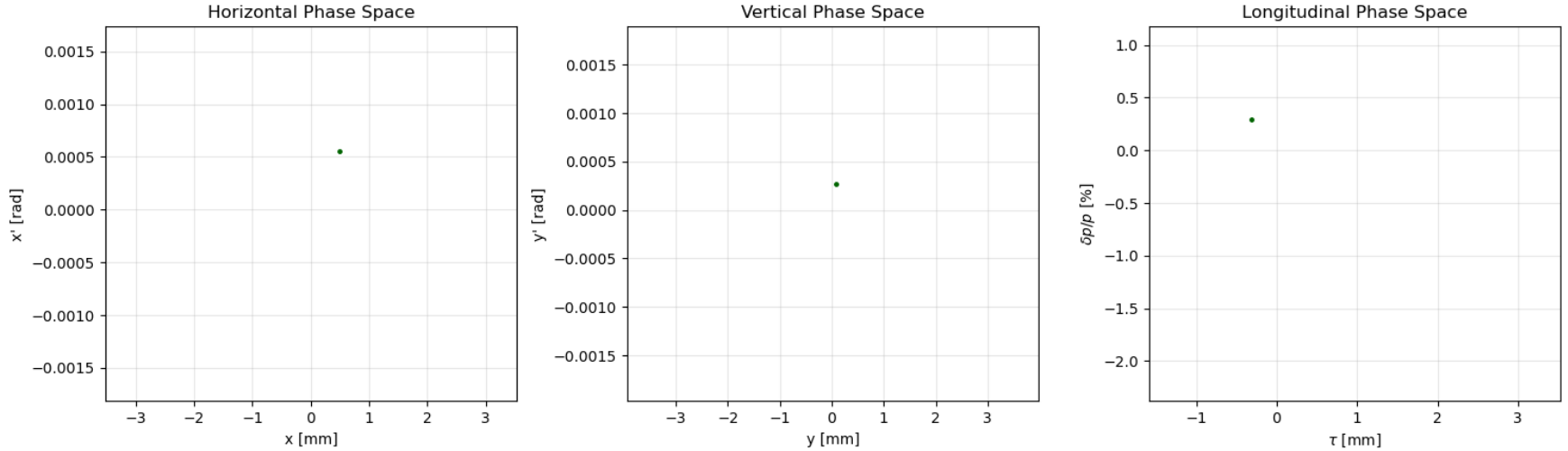


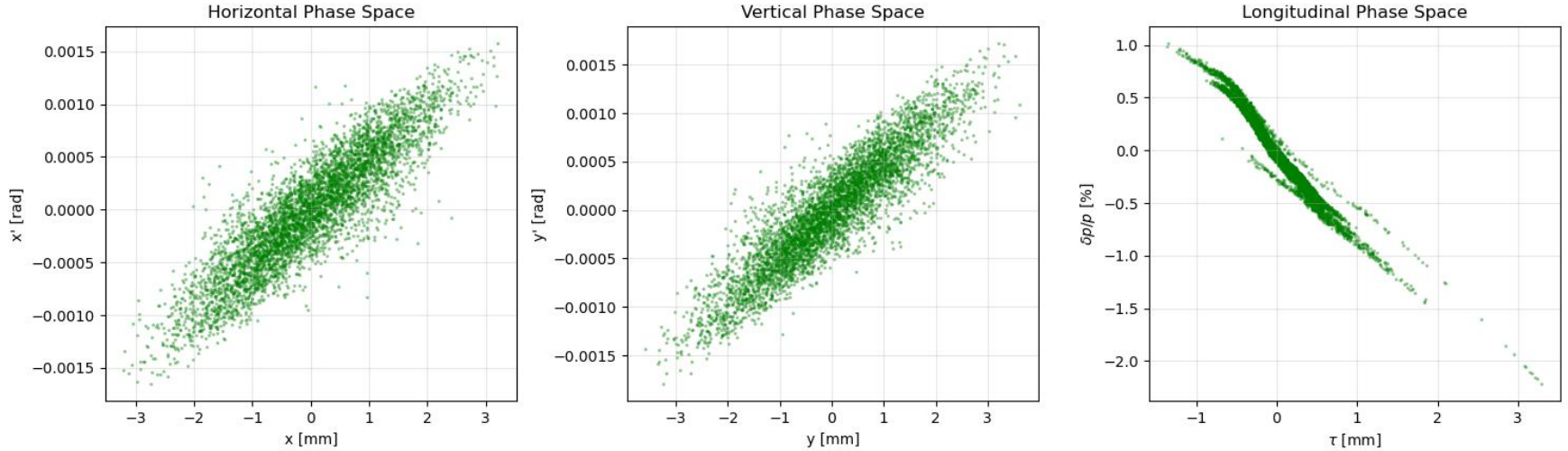
$$R_{total} = R_C R_B R_A$$

$$\mathbf{X}_{out} = R_{total}\mathbf{X}_{in}$$

Phase Space

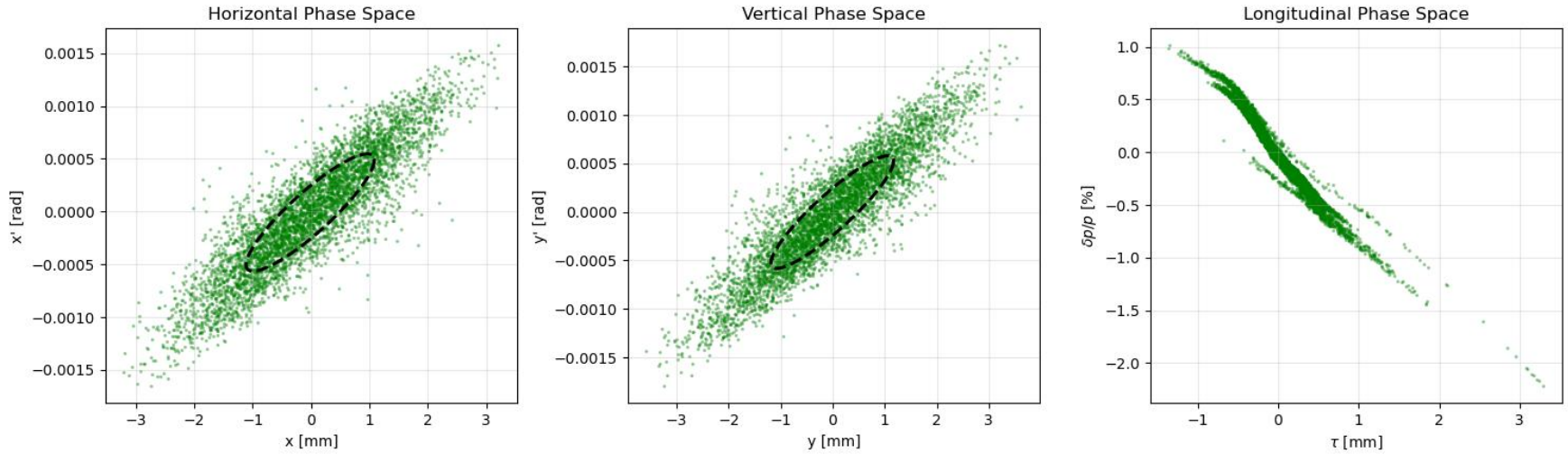
- A beam can be represented in phase space
- Each particle is represented by one point





Twiss Parameters

- The transverse distribution can be represented by an ellipse
- The phase-space ellipse is described by the emittance and Twiss parameters



Twiss Parameters

Emittance

Emittance determines the overall size of the phase-space ellipse.

$$\varepsilon = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2}$$

Beta

Beta describes the spatial width of the beam.

$$\langle x^2 \rangle = \varepsilon\beta$$

Alpha

Alpha describes the correlation between position

and trajectory slope and determines the tilt of the ellipse.

$$\langle xx' \rangle = -\varepsilon\alpha$$

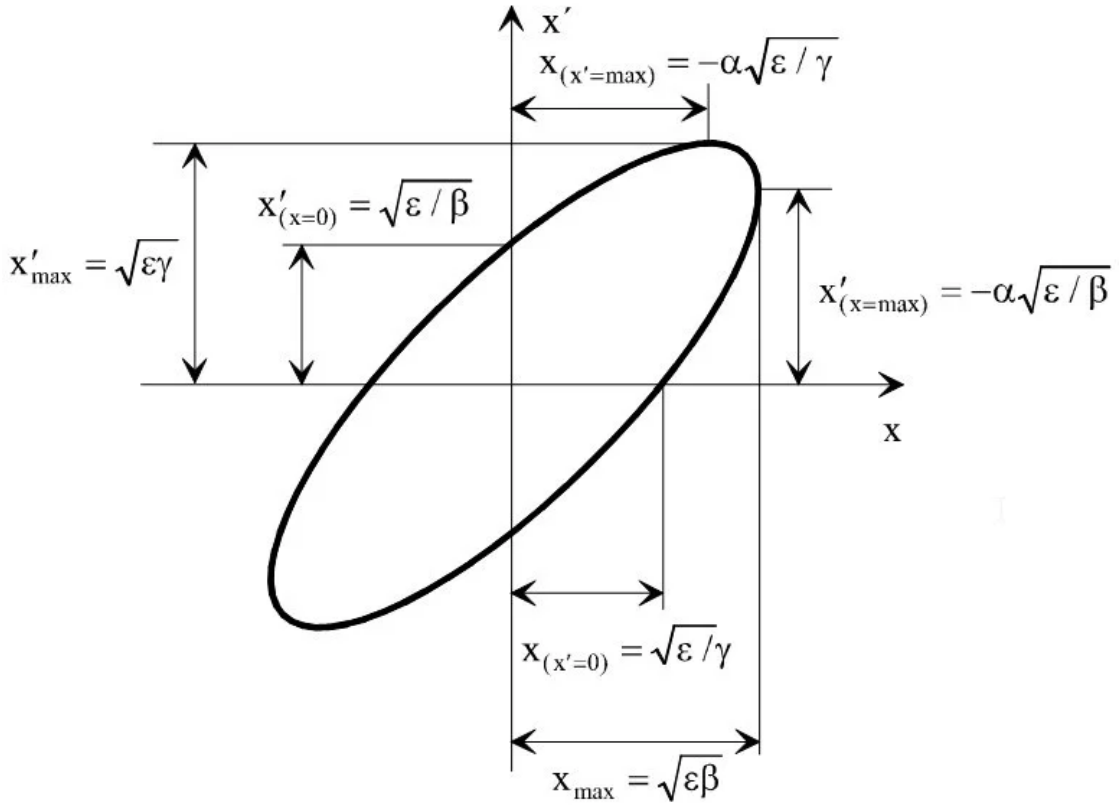
Gamma

Gamma describes the angular spread of the beam.

$$\langle x'^2 \rangle = \varepsilon\gamma$$

The Twiss parameters satisfy

$$\beta\gamma - \alpha^2 = 1 \qquad \gamma = \frac{1+\alpha^2}{\beta}$$



Beam Matrix

The phase-space distribution can be described by a beam matrix

$$\Sigma = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle & \langle xz \rangle & \langle x\delta \rangle \\ \langle x'x \rangle & \langle x'^2 \rangle & \langle x'y \rangle & \langle x'y' \rangle & \langle x'z \rangle & \langle x'\delta \rangle \\ \langle yx \rangle & \langle yx' \rangle & \langle y^2 \rangle & \langle yy' \rangle & \langle yz \rangle & \langle y\delta \rangle \\ \langle y'x \rangle & \langle y'x' \rangle & \langle y'y \rangle & \langle y'^2 \rangle & \langle y'z \rangle & \langle y'\delta \rangle \\ \langle zx \rangle & \langle zx' \rangle & \langle zy \rangle & \langle zy' \rangle & \langle z^2 \rangle & \langle z\delta \rangle \\ \langle \delta x \rangle & \langle \delta x' \rangle & \langle \delta y \rangle & \langle \delta y' \rangle & \langle \delta z \rangle & \langle \delta^2 \rangle \end{pmatrix}$$

Beam Reconstruction

We can treat each block of the matrix separately for independent analysis when the beam is not coupled.

$$\sigma = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle \\ \langle xx' \rangle & \langle x'^2 \rangle \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$\sqrt{\det(\sigma)} = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{21}^2}$$

$$\det(\sigma) = \det\left(\varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}\right) = \varepsilon^2$$

$$\det(\sigma) = \varepsilon^2(\beta\gamma - \alpha^2) \qquad \gamma = \frac{1+\alpha^2}{\beta}$$

$$\sqrt{\det(\sigma)} = \sqrt{\varepsilon^2} = \varepsilon$$

$$\varepsilon_x = \sqrt{\sigma_x^2\sigma_{x'}^2 - \sigma_{xx'}^2}$$

$$\Sigma_f = R\Sigma_i R^T$$

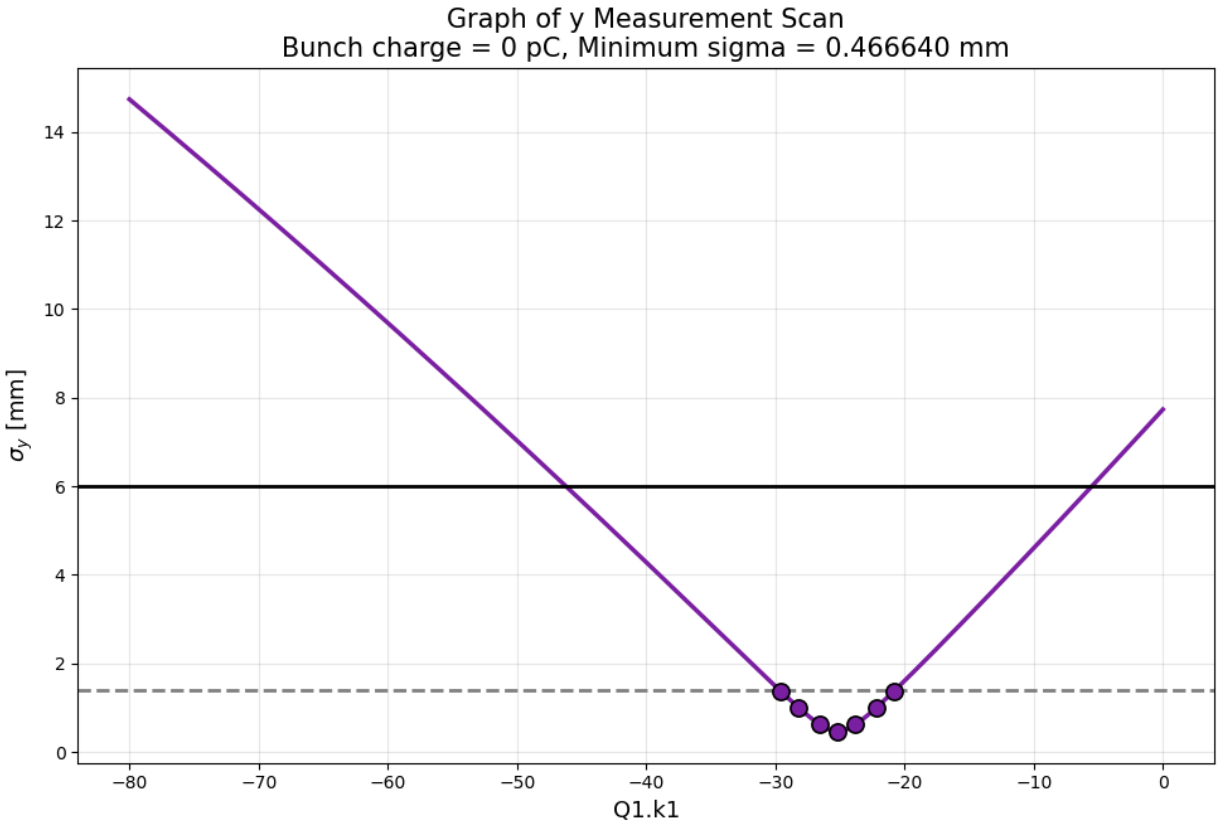
the beam matrix is transported according to this equation

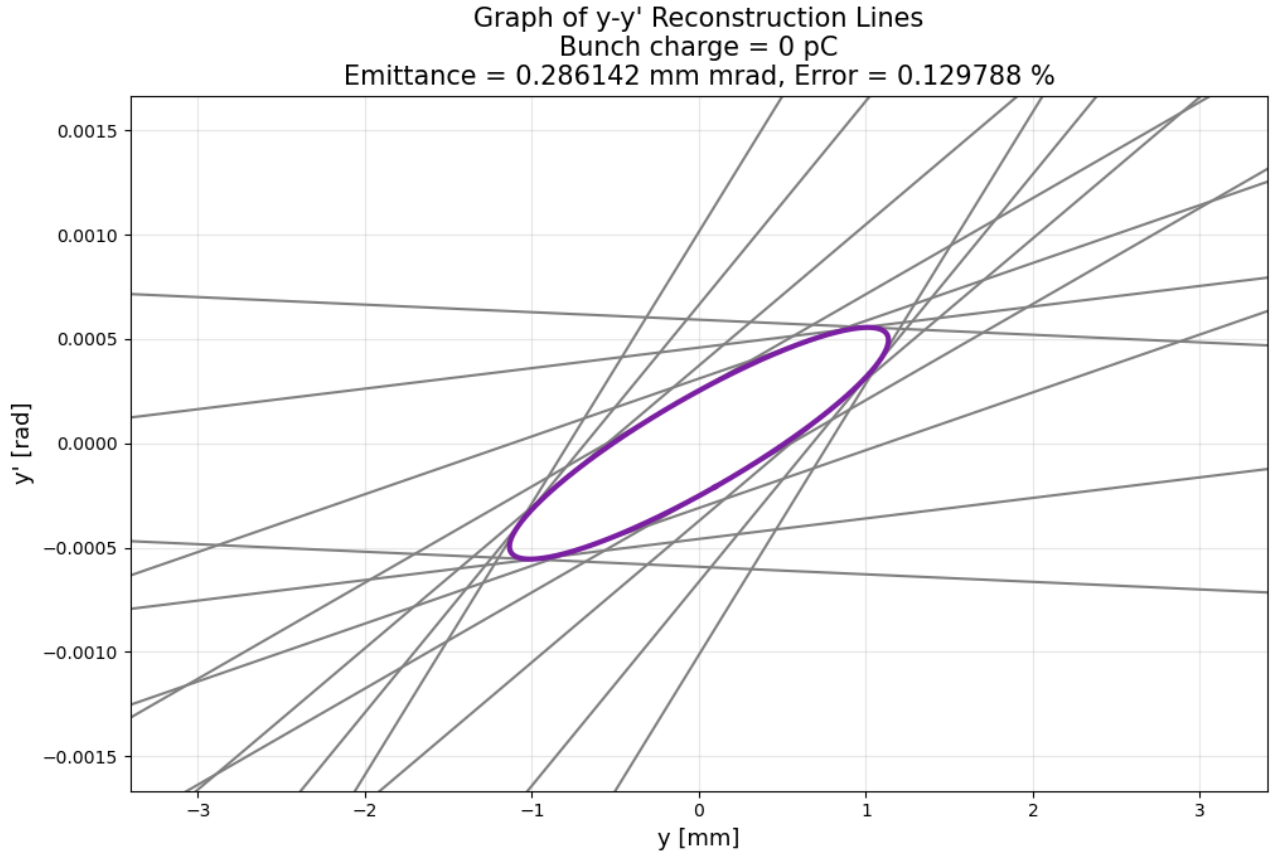
$$\beta_x = \frac{\sigma_x^2}{\varepsilon_x}$$

$$\sigma_{x,f}^2 = R_{11}^2\sigma_{x,i}^2 + 2R_{11}R_{12}\sigma_{xx',i} + R_{12}^2\sigma_{x',i}^2$$

$$\alpha_x = -\frac{\sigma_{xx'}}{\varepsilon_x}$$

rtunc@tarla-fel.org



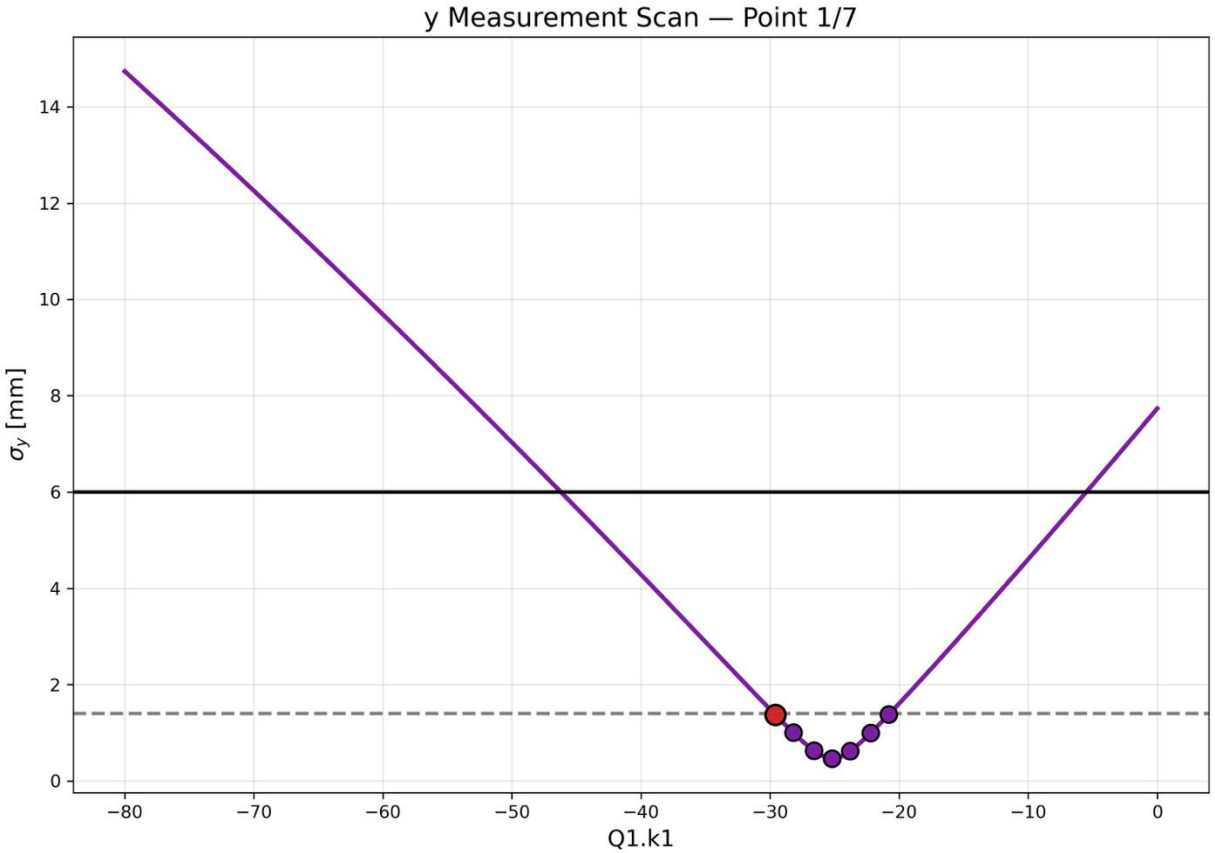


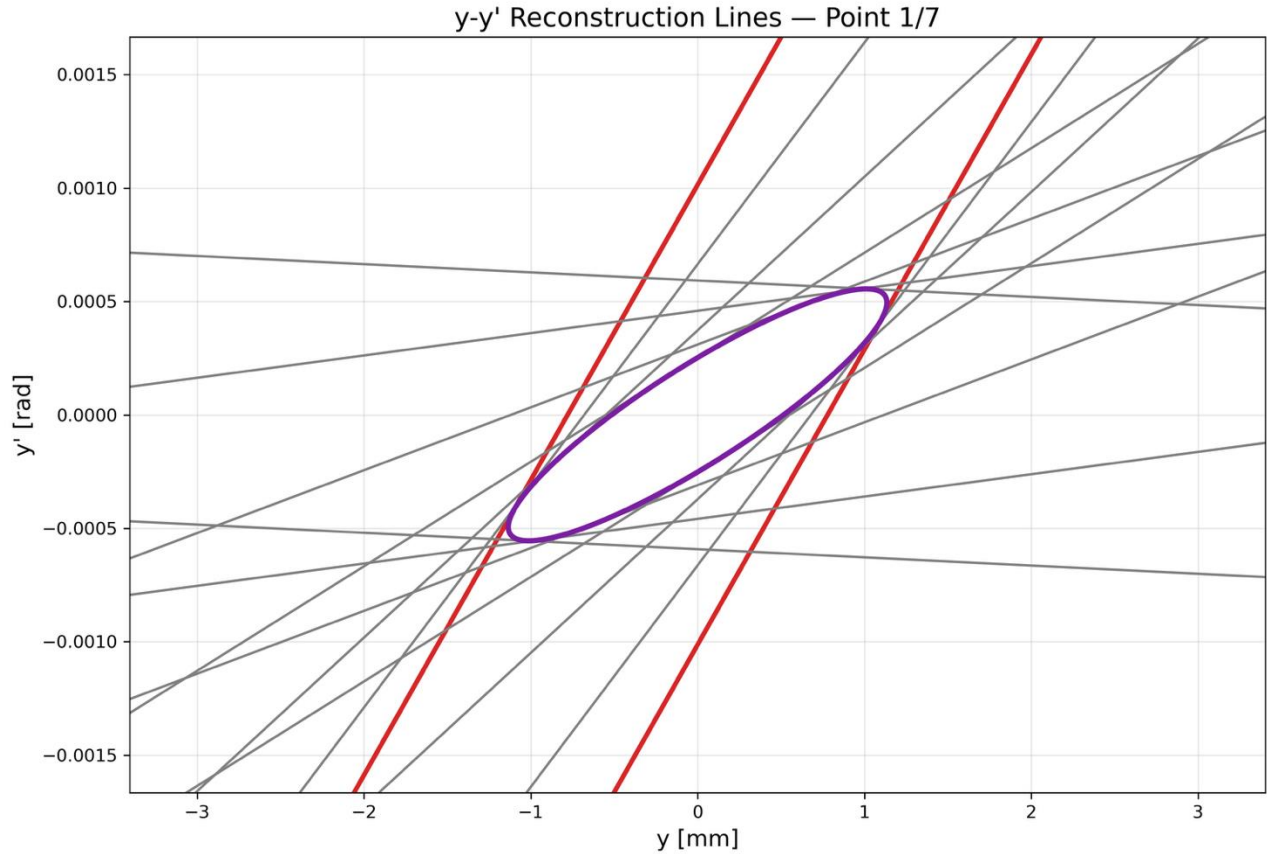
$$x_f = R_{11}x_0 + R_{12}x_0'$$

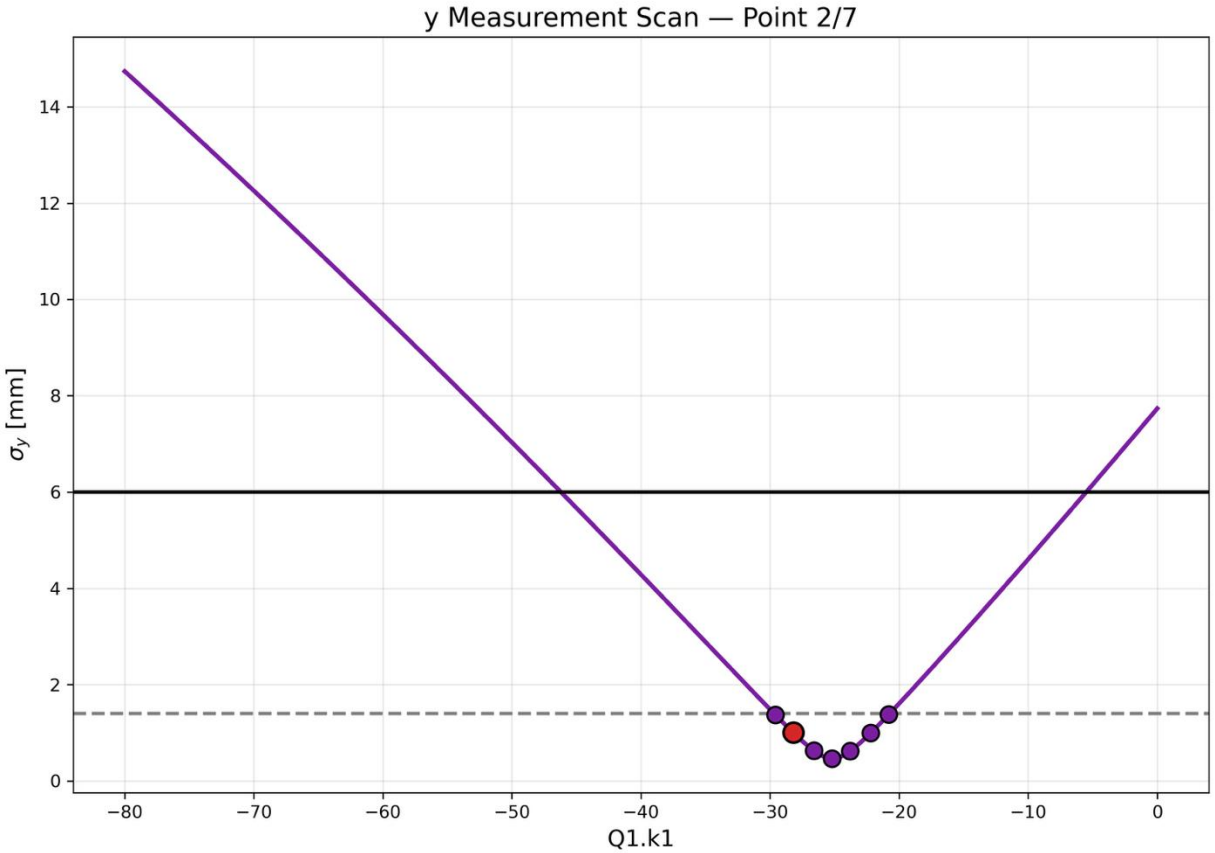
$$R_{11}x_0 + R_{12}x_0' = \pm\sigma_x$$

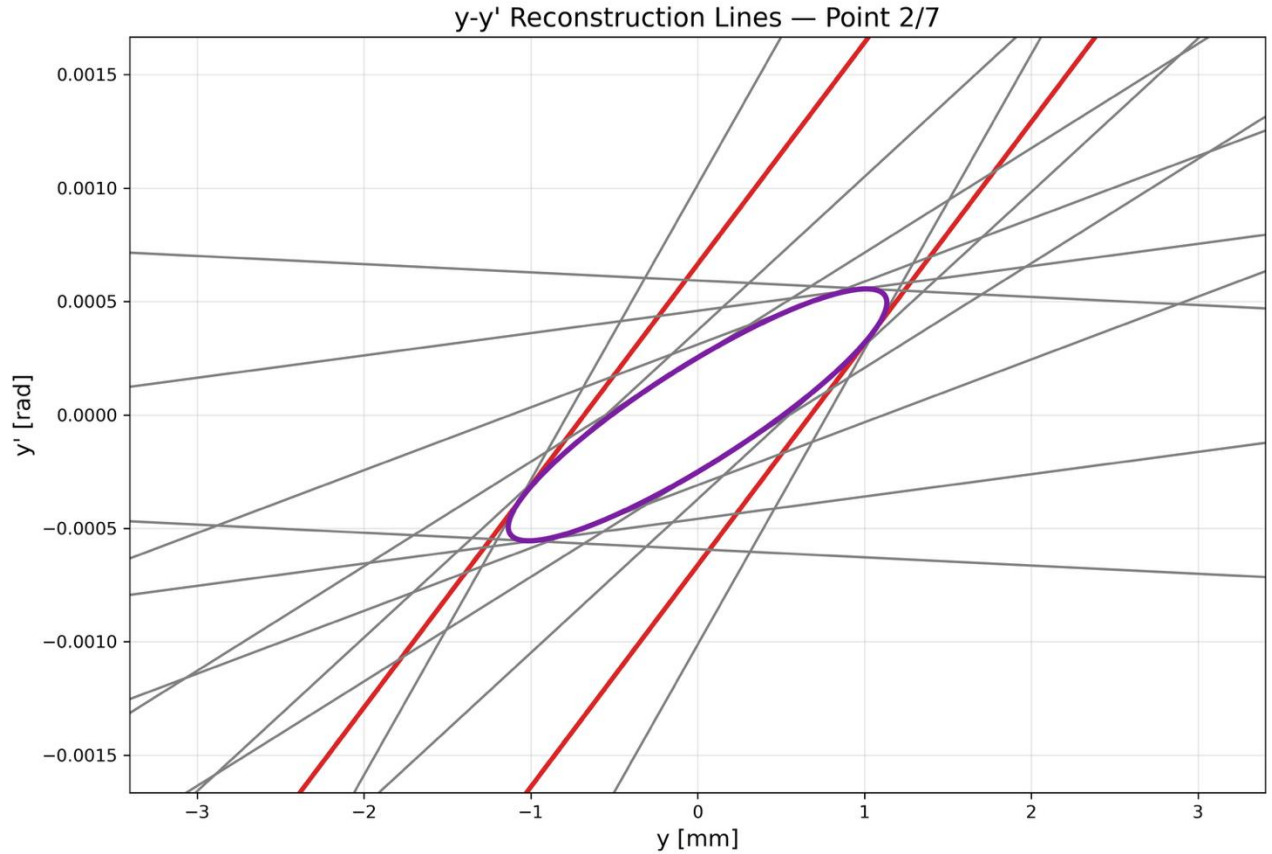
$$x_0' = \frac{\sigma_x - R_{11}x_0}{R_{12}}$$

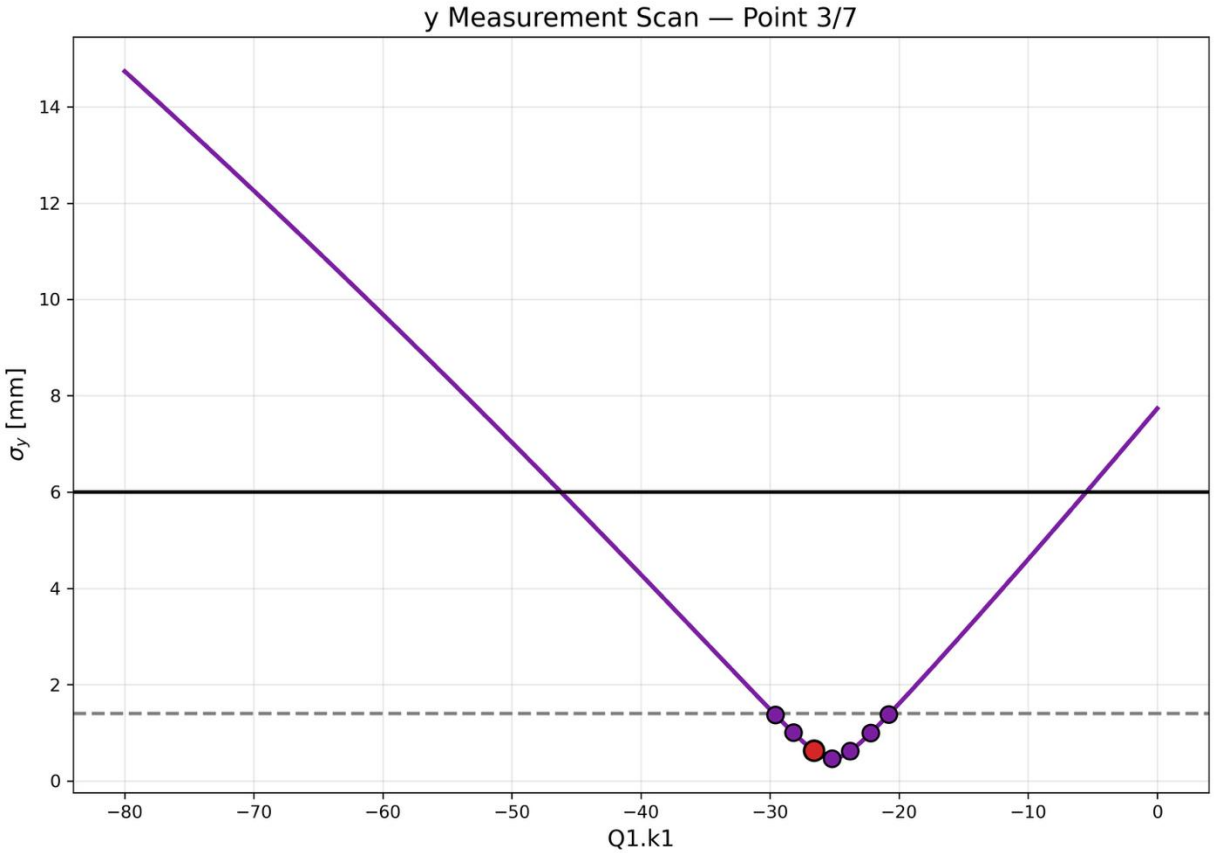
$$x_0' = \frac{-\sigma_x - R_{11}x_0}{R_{12}}$$


y Measurement Scan — Point 1/7
σ_y [mm]
Q1.k1
14
12
10
8
6
4
2
0
−80
−70
−60
−50
−40
−30
−20
−10
0


y-y' Reconstruction Lines — Point 1/7
y' [rad]
y [mm]
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
−0.0015
−3
−2
−1
0
1
2
3


y Measurement Scan — Point 2/7
σ_y [mm]
Q1.k1
14
12
10
8
6
4
2
0
−80
−70
−60
−50
−40
−30
−20
−10
0


y-y' Reconstruction Lines — Point 2/7
y' [rad]
y [mm]
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
−0.0015
−3
−2
−1
0
1
2
3


y Measurement Scan — Point 3/7
σ_y [mm]
Q1.k1
14
12
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8
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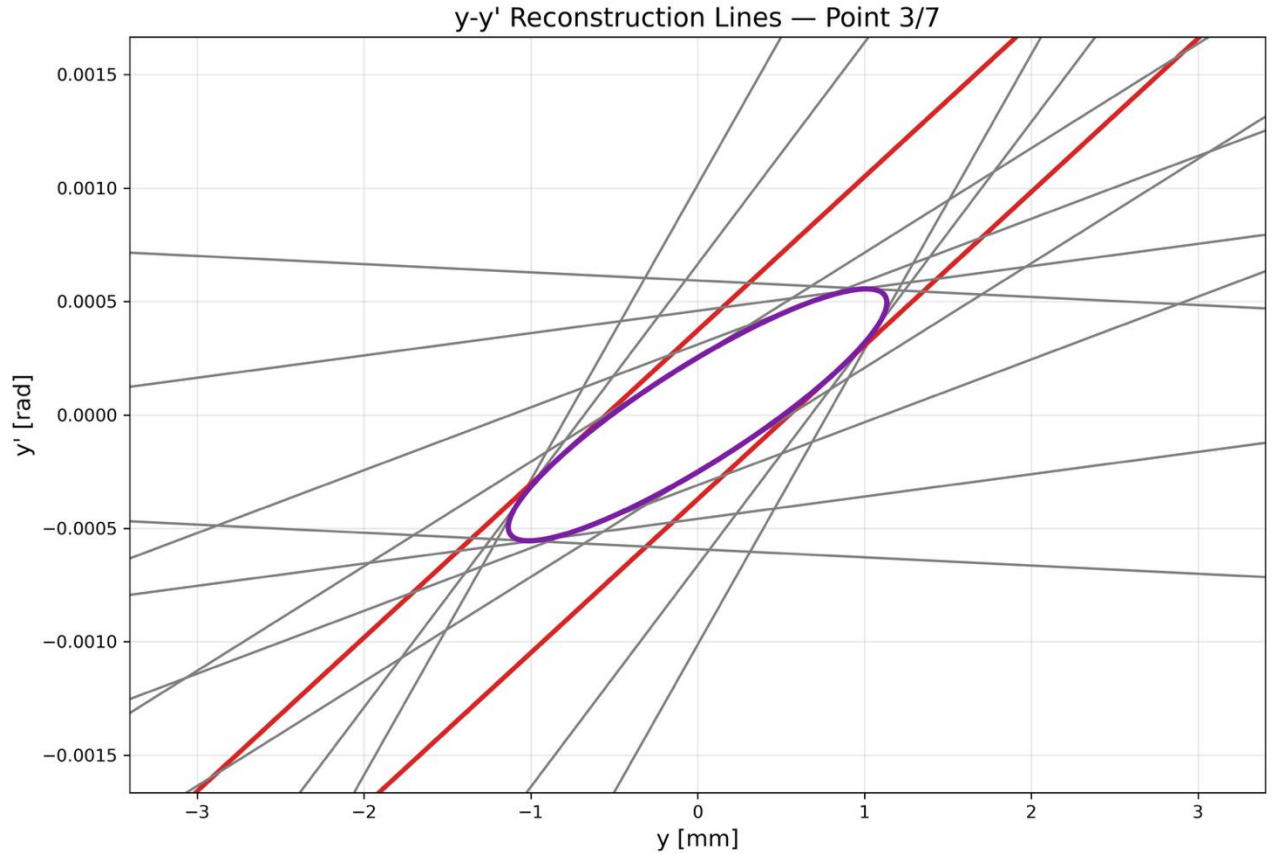
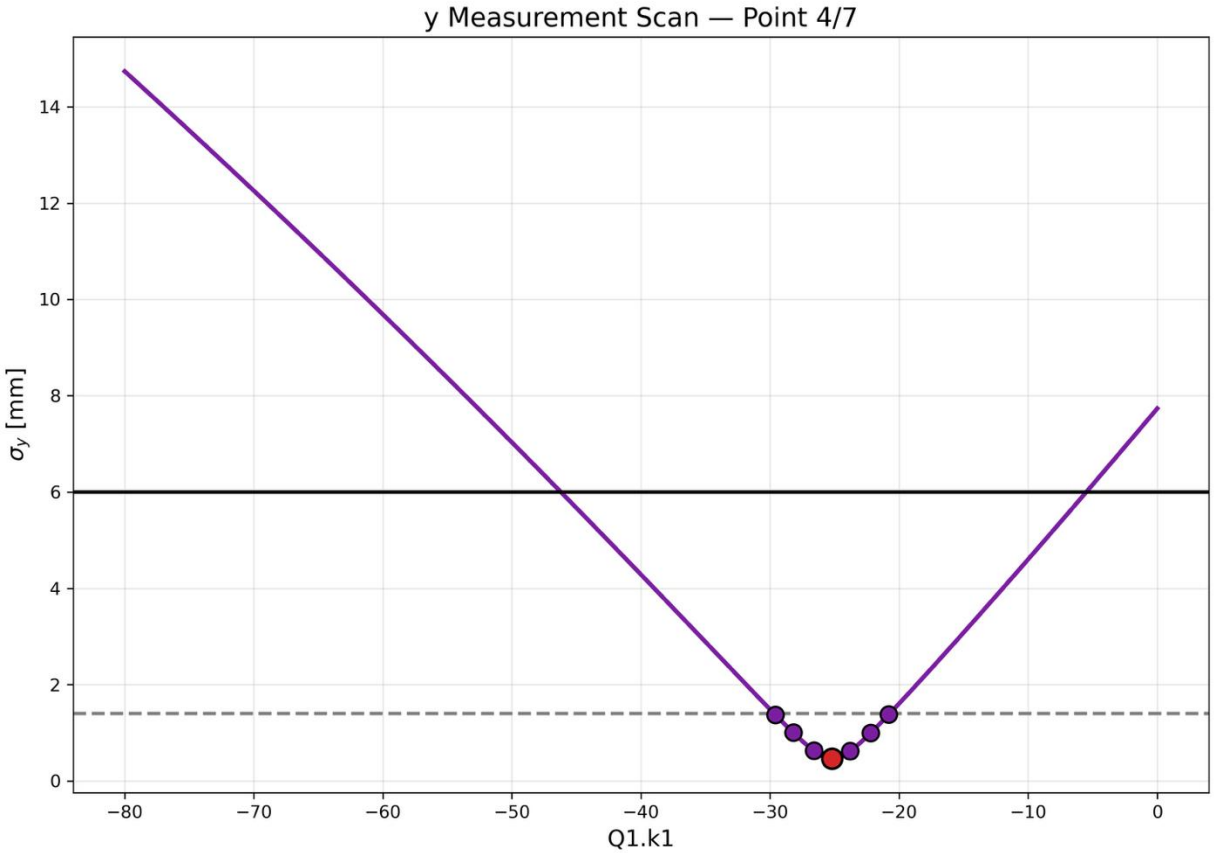
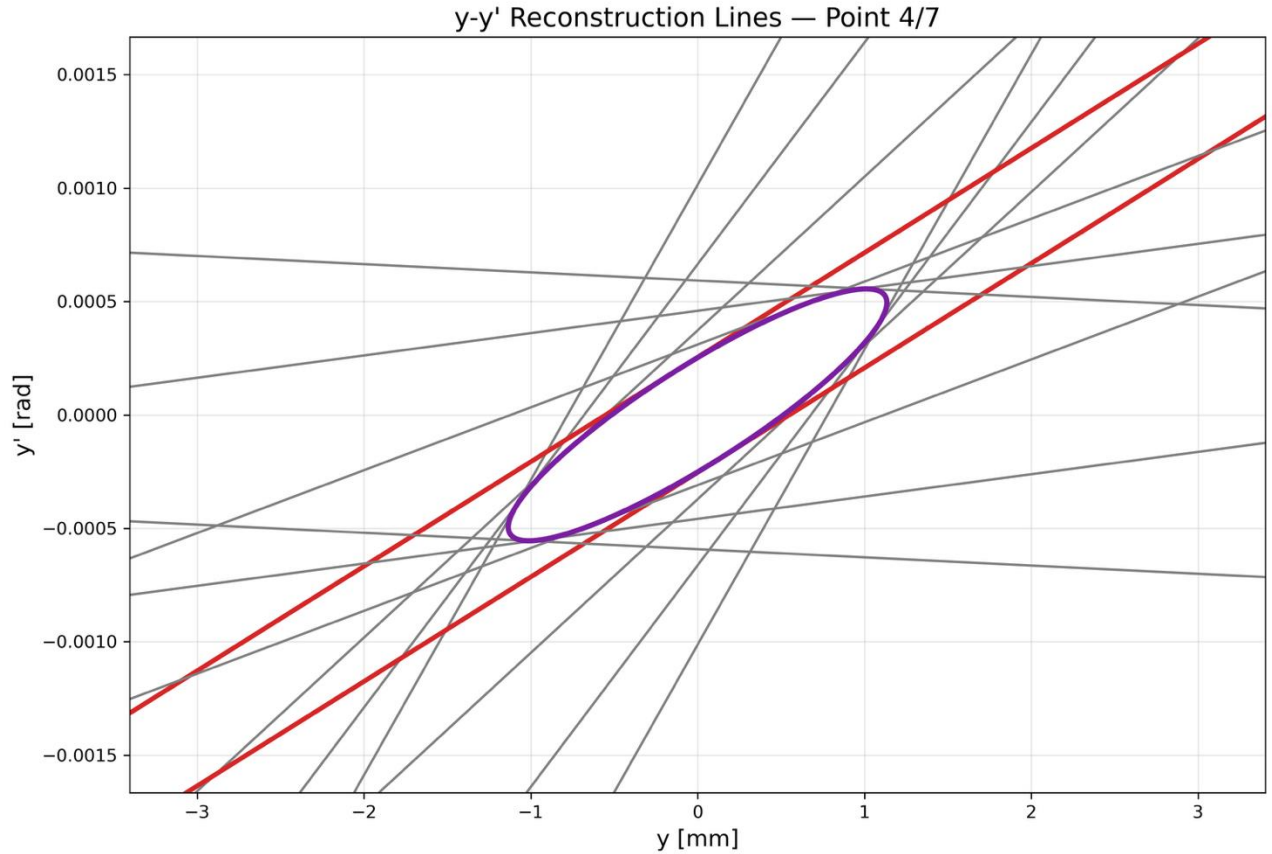
−50
−40
−30
−20
−10
0


y-y' Reconstruction Lines — Point 3/7
y' [rad]
y [mm]
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
−0.0015
−3
−2
−1
0
1
2
3


y Measurement Scan — Point 4/7
σ_y [mm]
Q1.k1
14
12
10
8
6
4
2
0
−80
−70
−60
−50
−40
−30
−20
−10
0


y-y' Reconstruction Lines — Point 4/7
y' [rad]
y [mm]
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
−0.0015
−3
−2
−1
0
1
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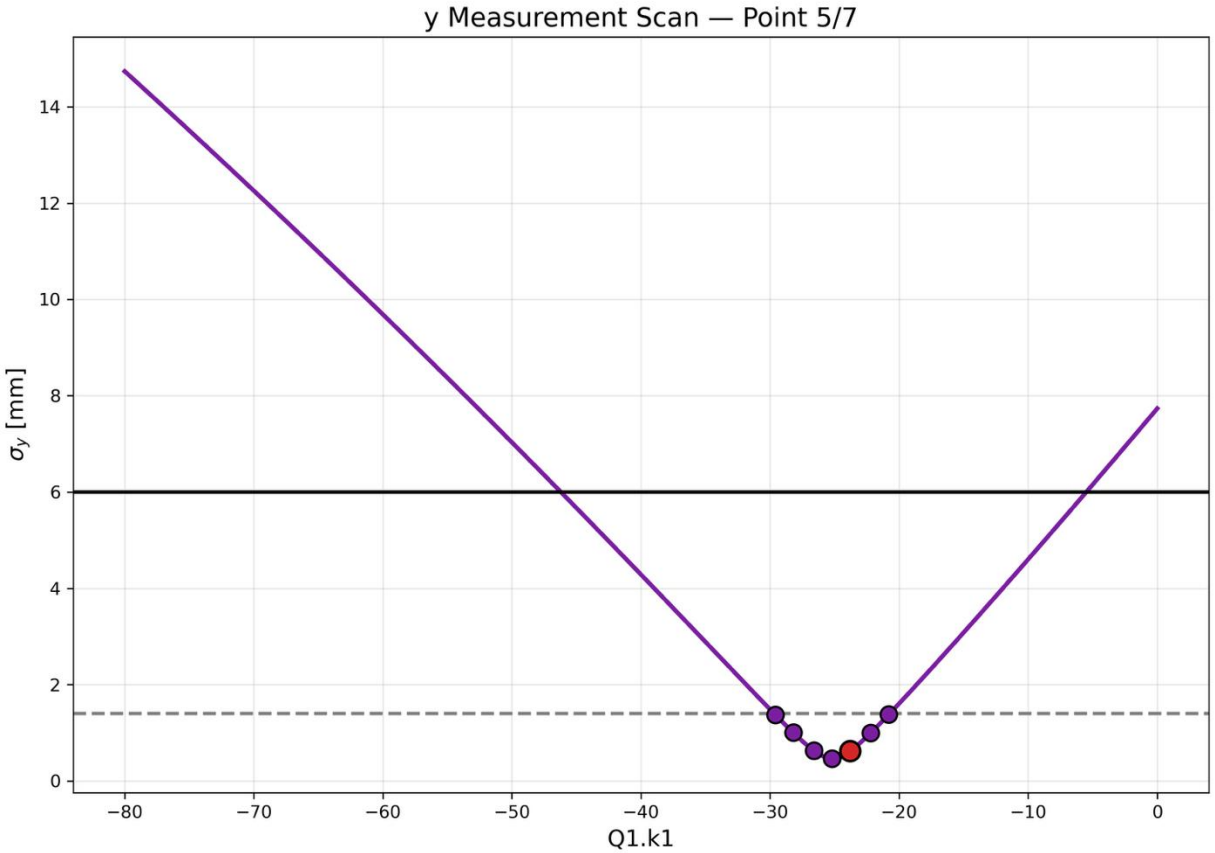
y Measurement Scan — Point 5/7
σ_y [mm]
Q1.k1
14
12
10
8
6
4
2
0
−80
−70
−60
−50
−40
−30
−20
−10
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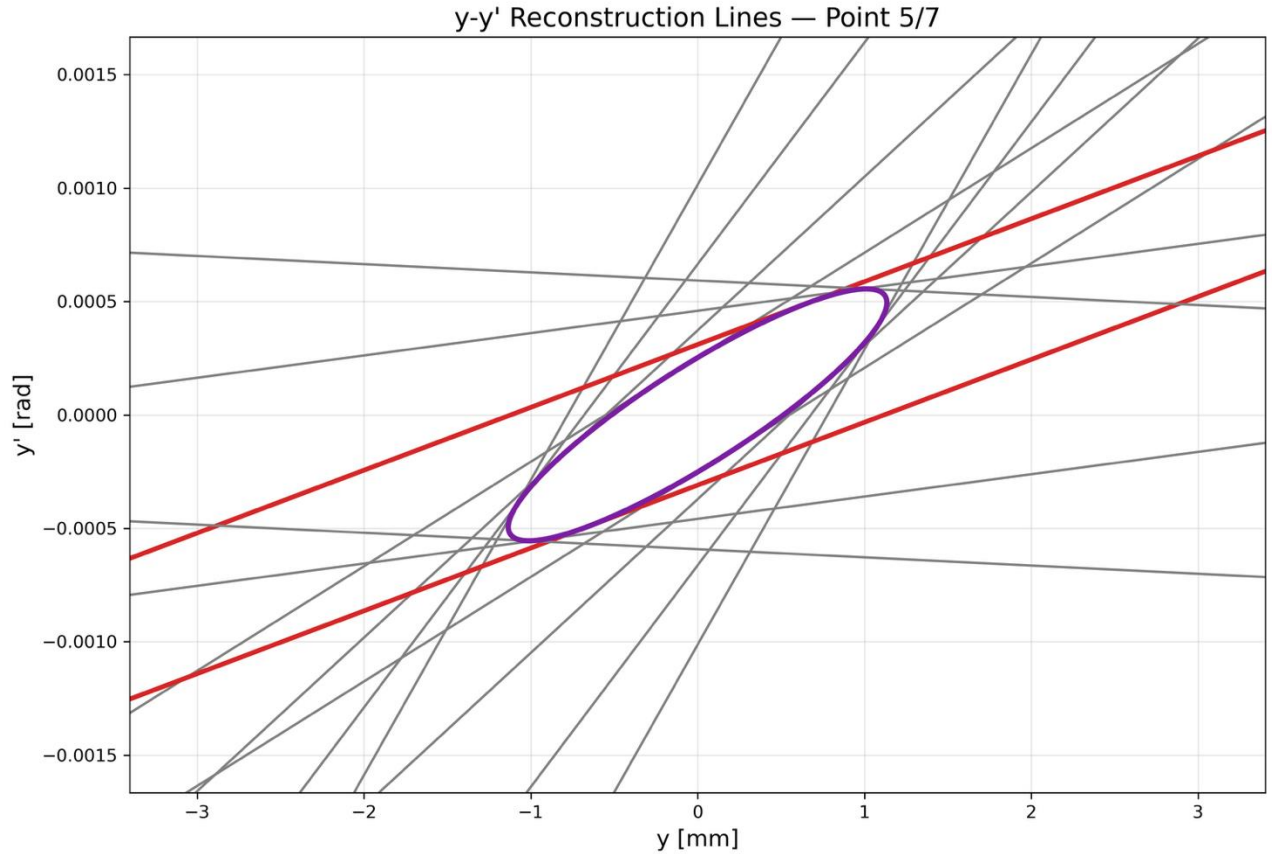
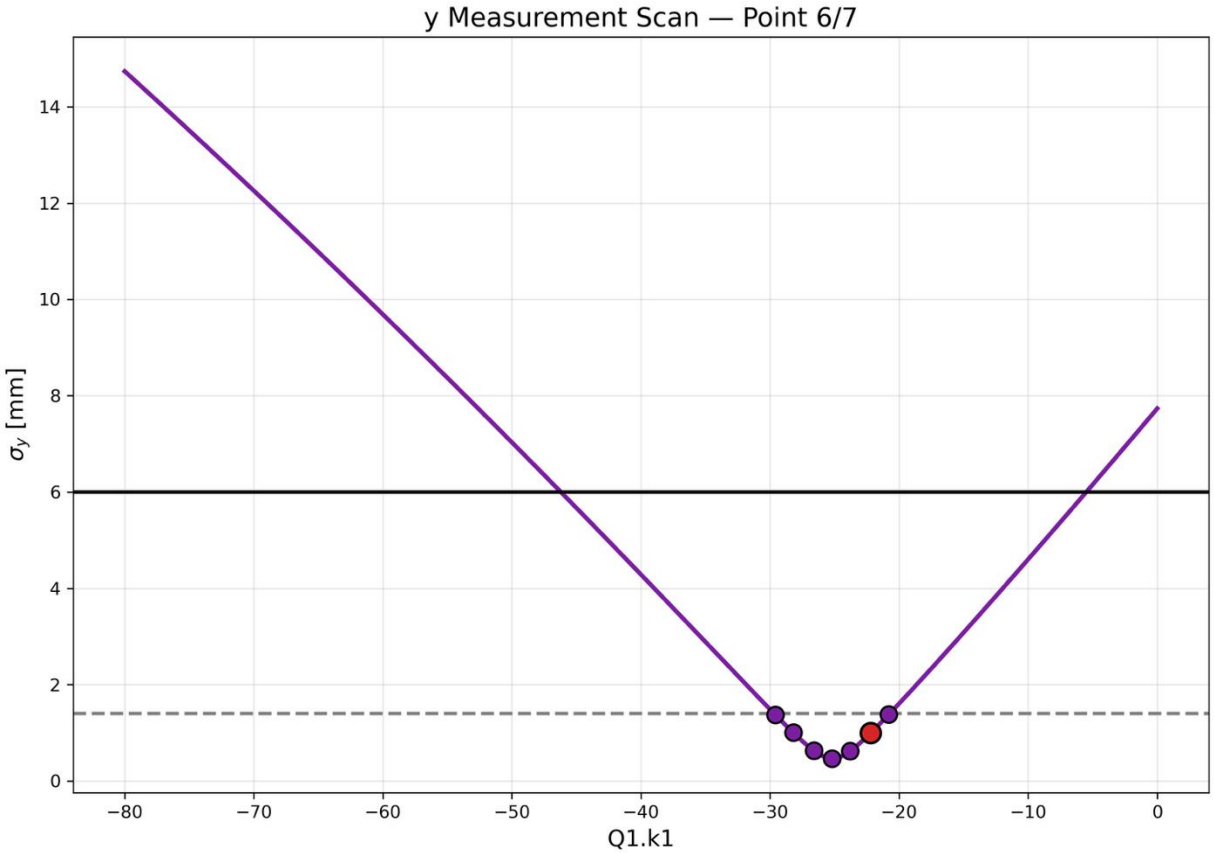

y-y' Reconstruction Lines — Point 5/7
y' [rad]
y [mm]
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
−0.0015
−3
−2
−1
0
1
2
3


y Measurement Scan — Point 6/7
σ_y [mm]
Q1.k1
14
12
10
8
6
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2
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−80
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−50
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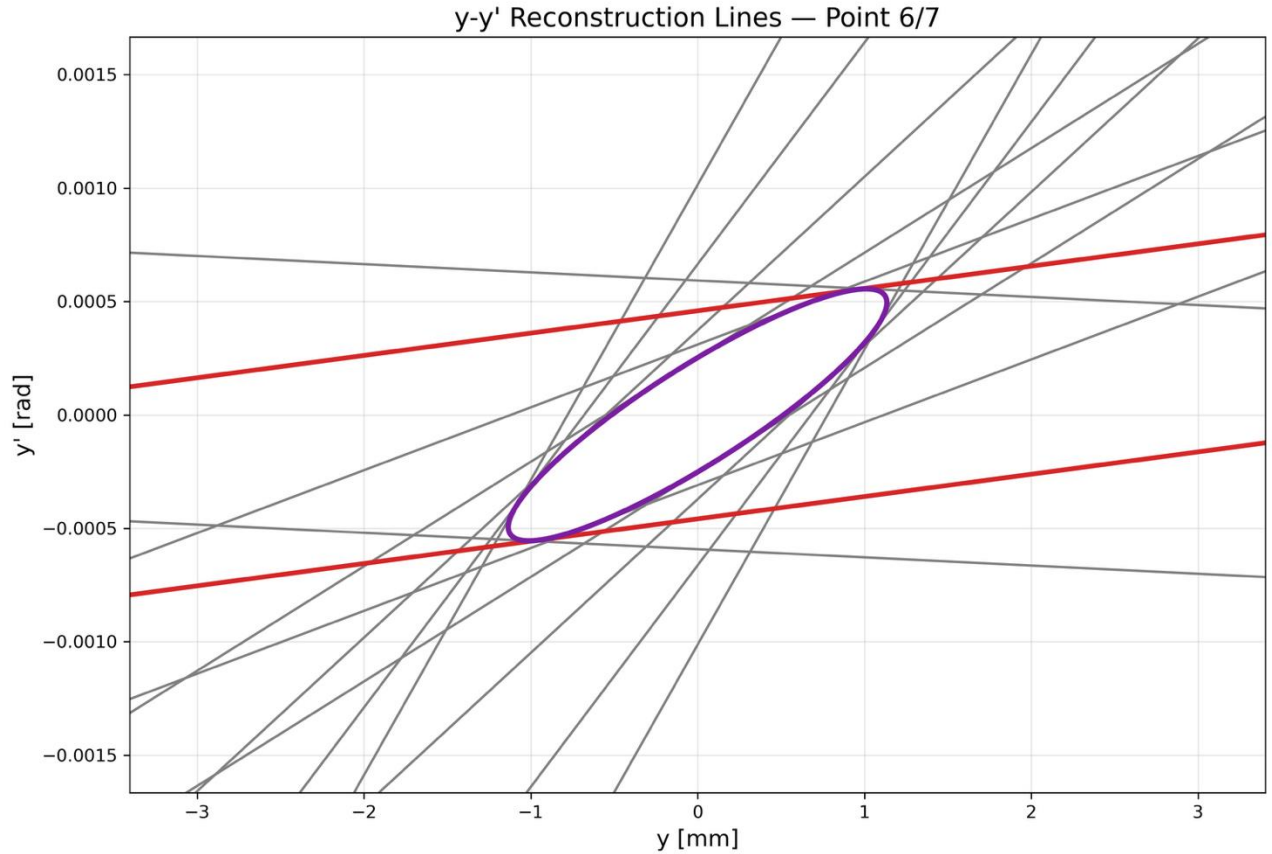
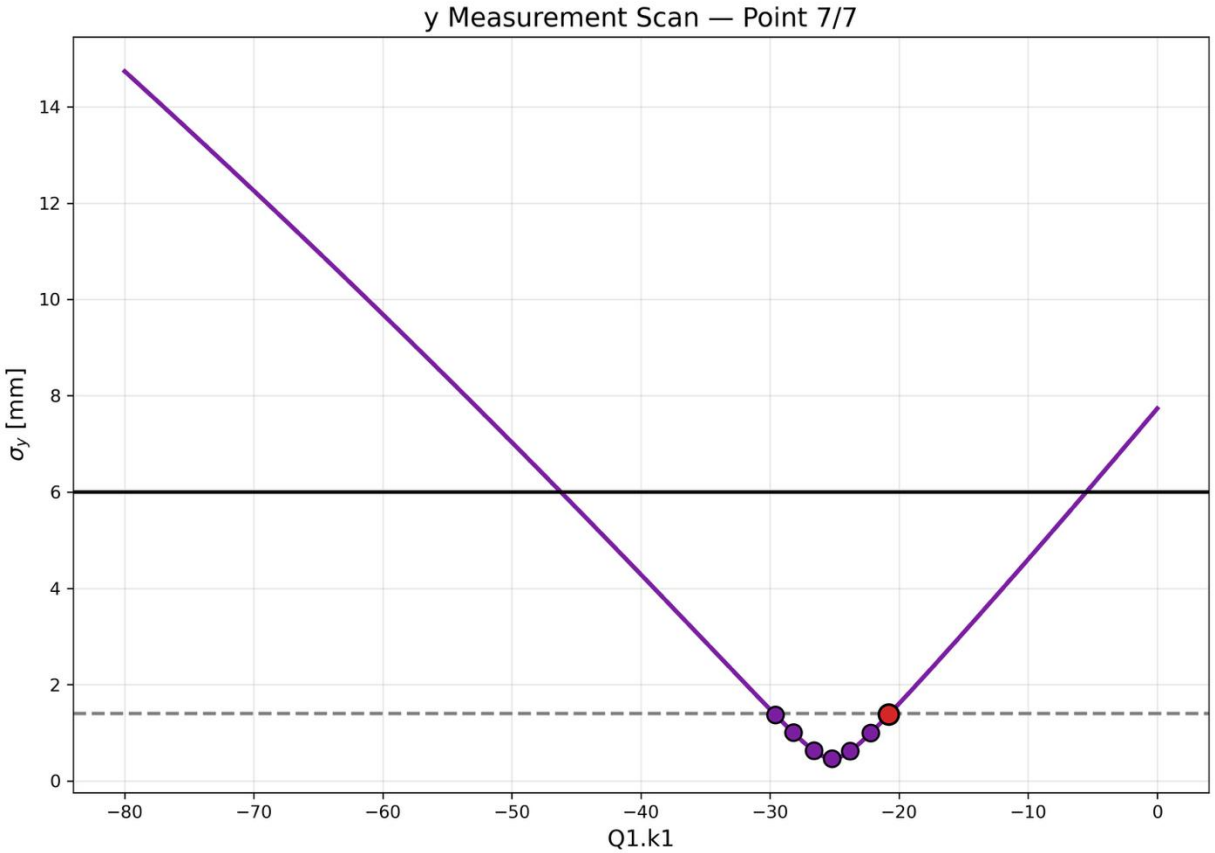
−30
−20
−10
0


y-y' Reconstruction Lines — Point 6/7
y' [rad]
y [mm]
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
−0.0015
−3
−2
−1
0
1
2
3


y Measurement Scan — Point 7/7
σ_y [mm]
Q1.k1
14
12
10
8
6
4
2
0
−80
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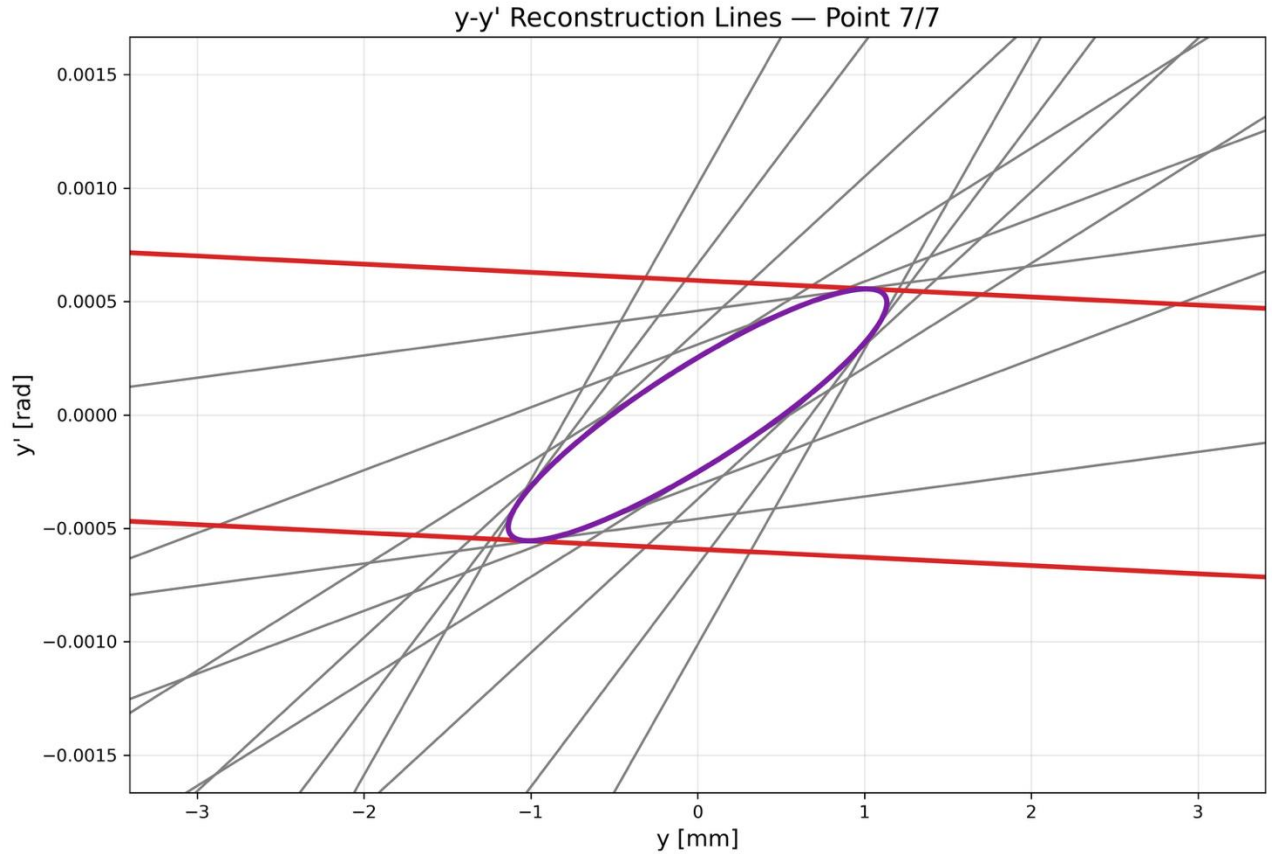
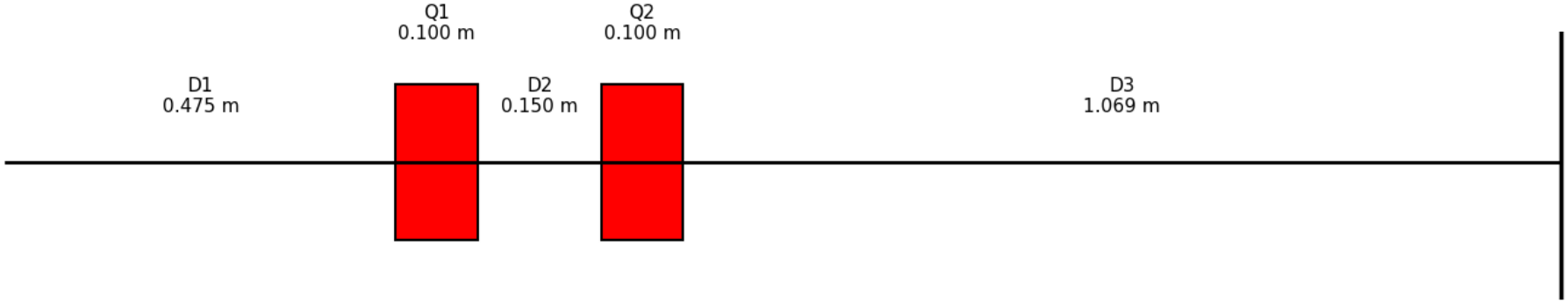
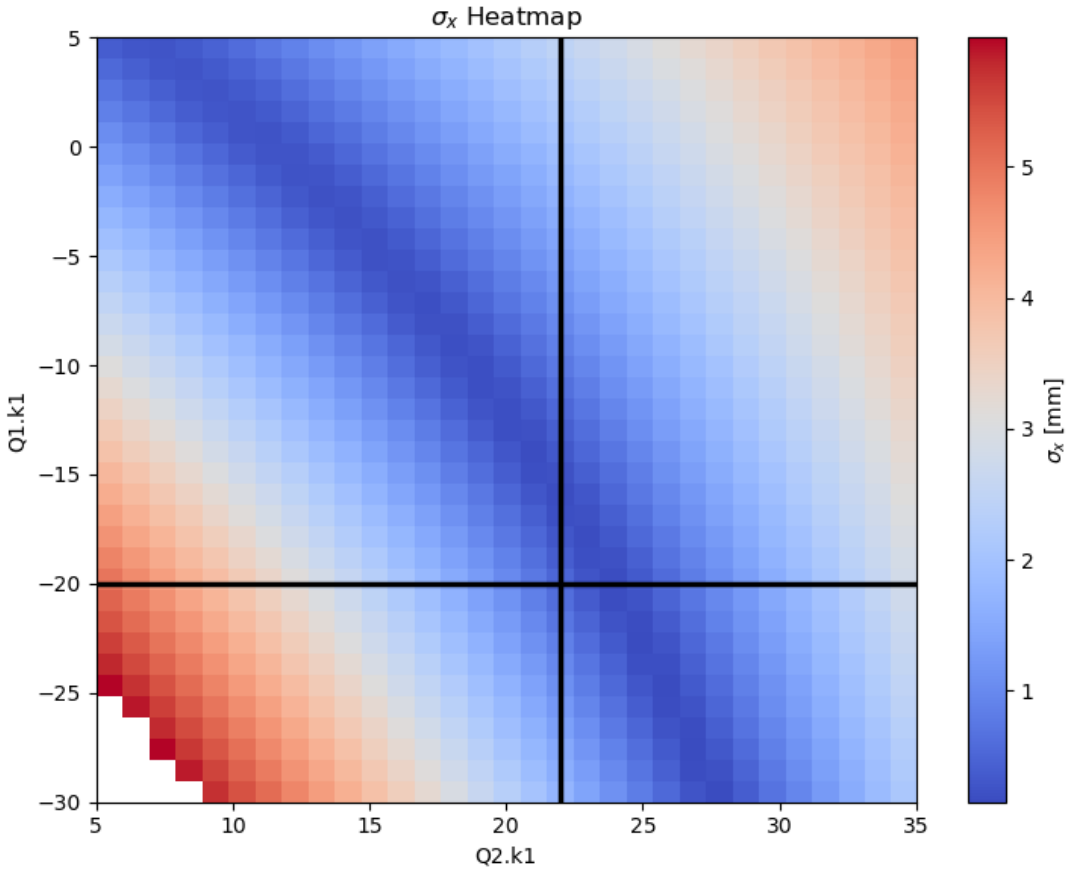
−50
−40
−30
−20
−10
0

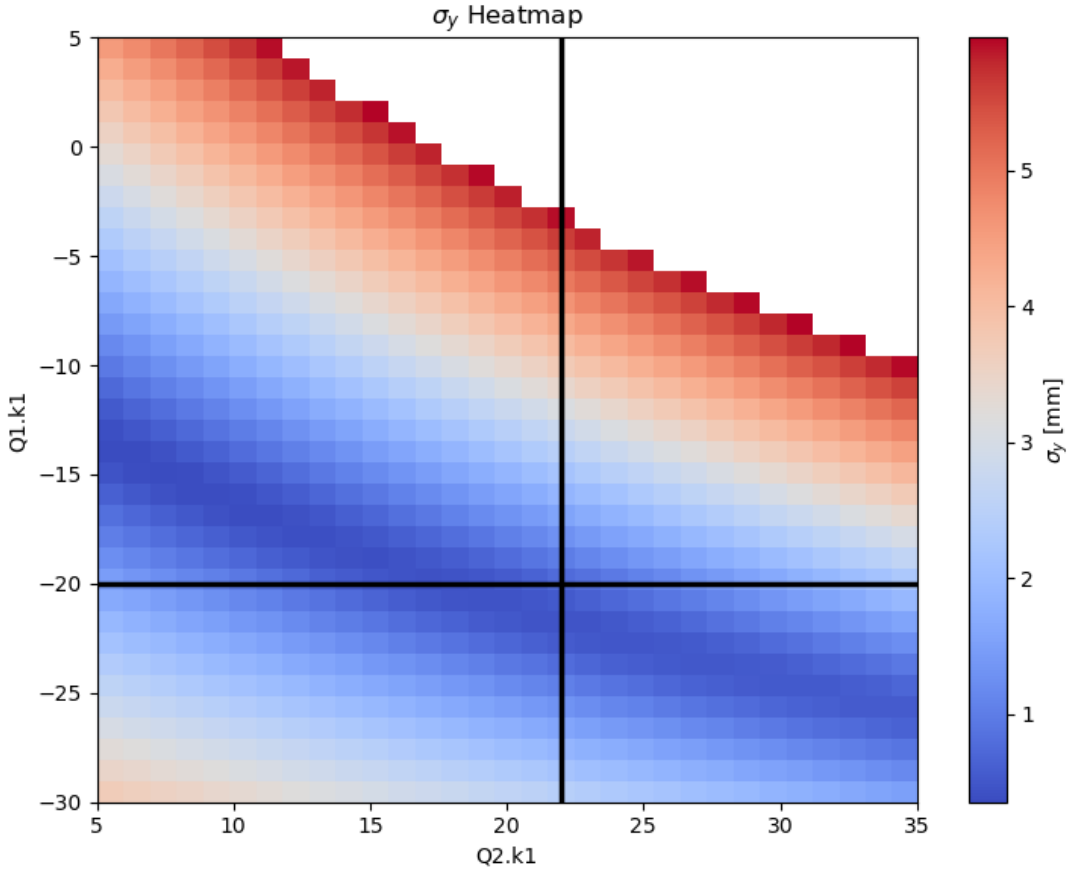

y-y' Reconstruction Lines — Point 7/7
y' [rad]
y [mm]
0.0015
0.0010
0.0005
0.0000
−0.0005
−0.0010
−0.0015
−3
−2
−1
0
1
2
3

BEAMLINE



HEATMAPS

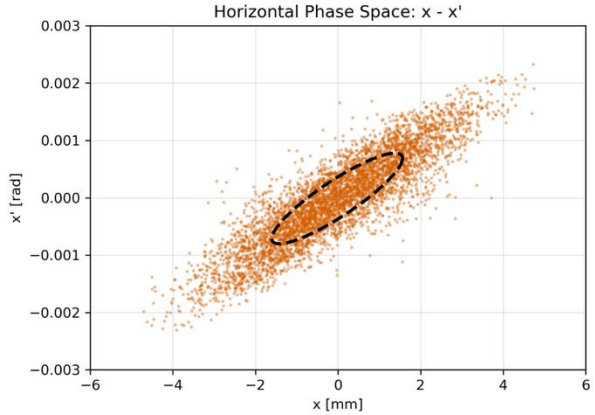


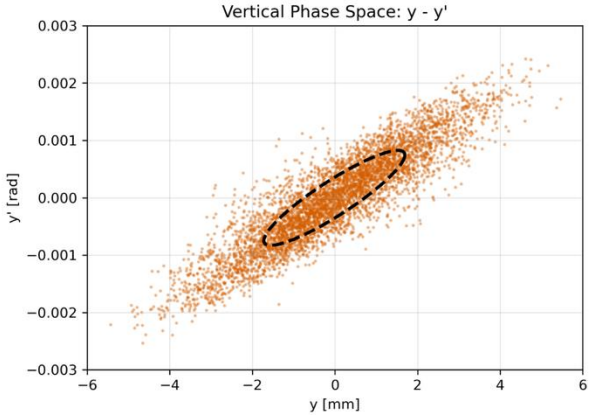


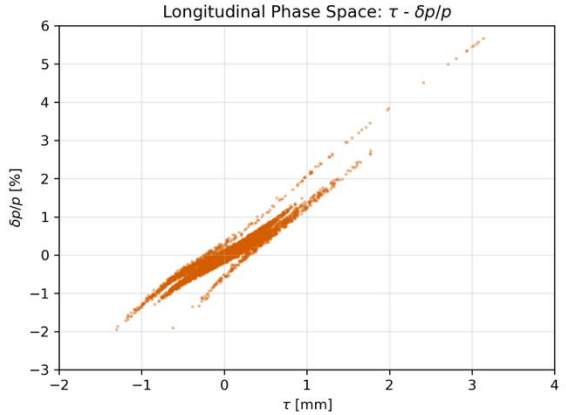
First Quadrupole: k = -20
Second Quadrupole: k = 22

BEAM FILES

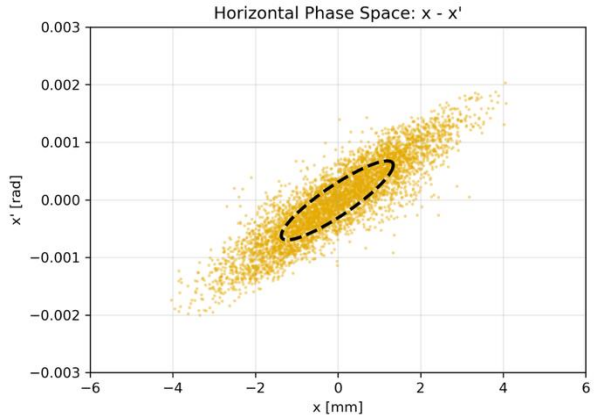
Initial Phase Space — 8.37 MeV, phase = -90°

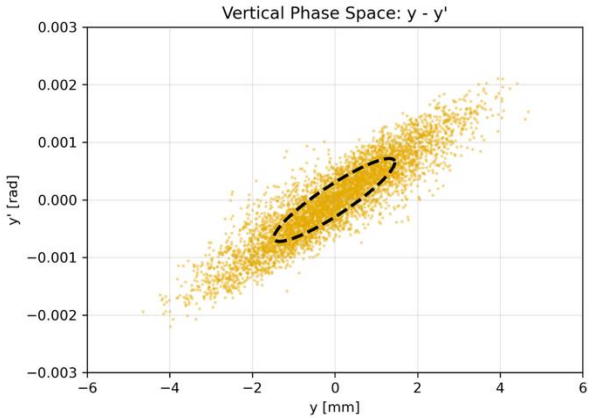


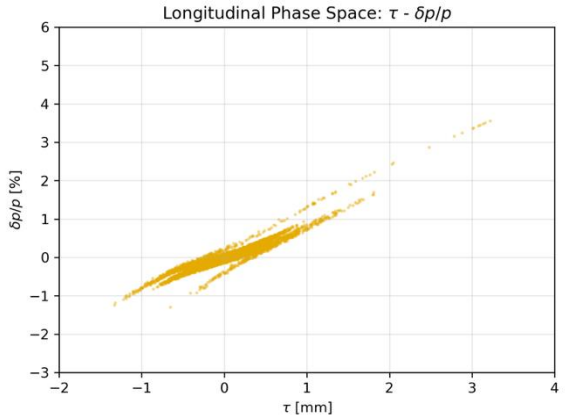


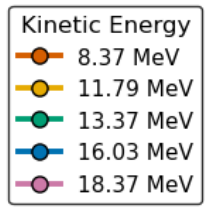


Initial Phase Space — 11.79 MeV, phase = -70°

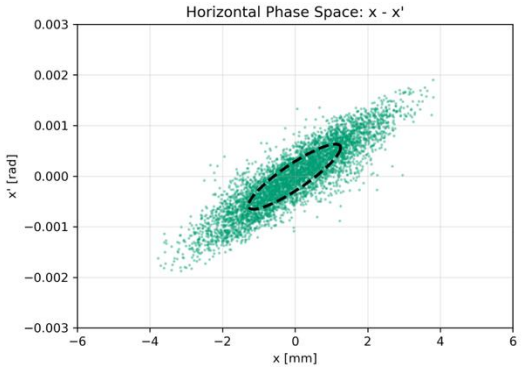


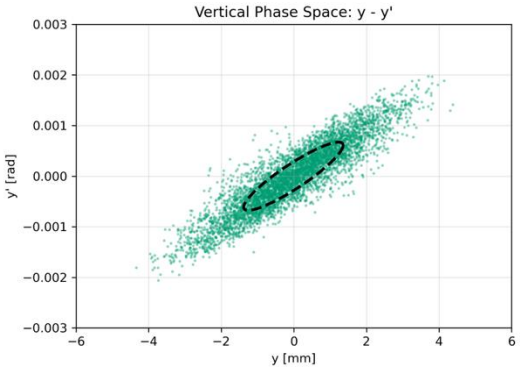


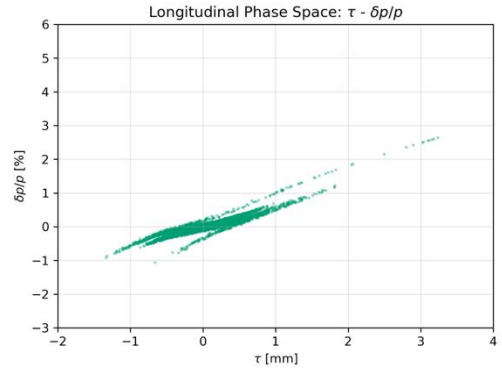




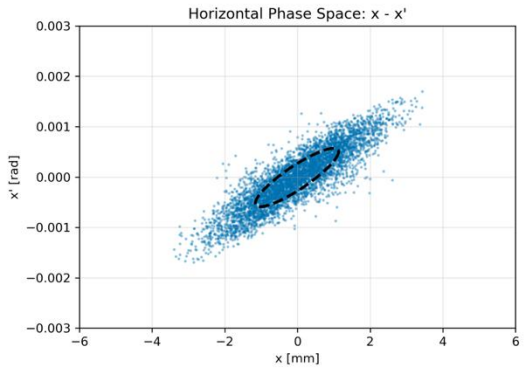
Initial Phase Space — 13.37 MeV, phase = -60°

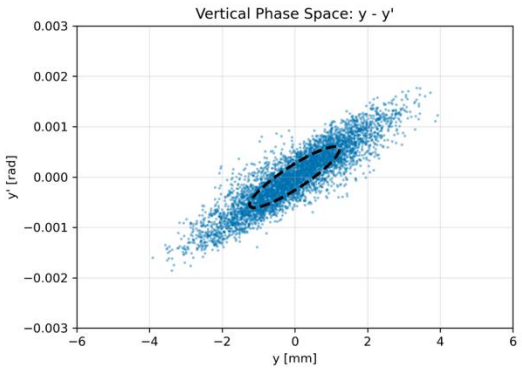


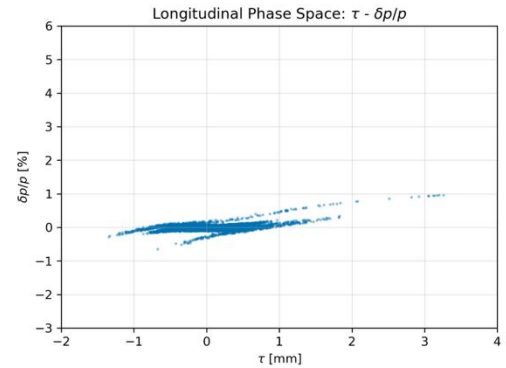




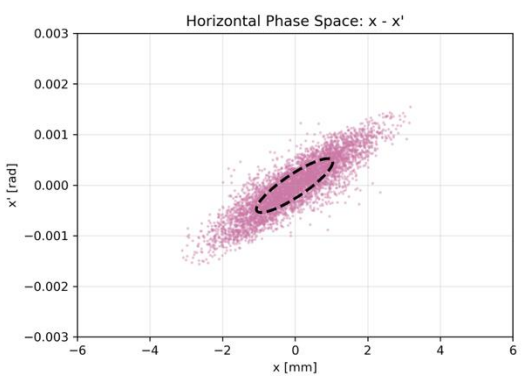
Initial Phase Space — 16.03 MeV, phase = -40°

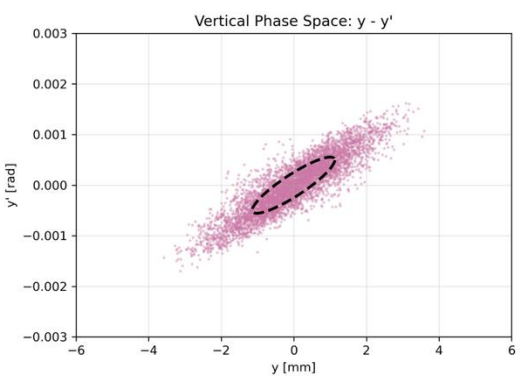


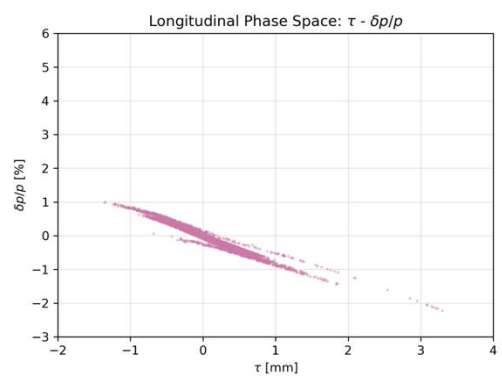


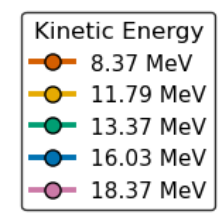


Initial Phase Space — 18.37 MeV, phase = 0°





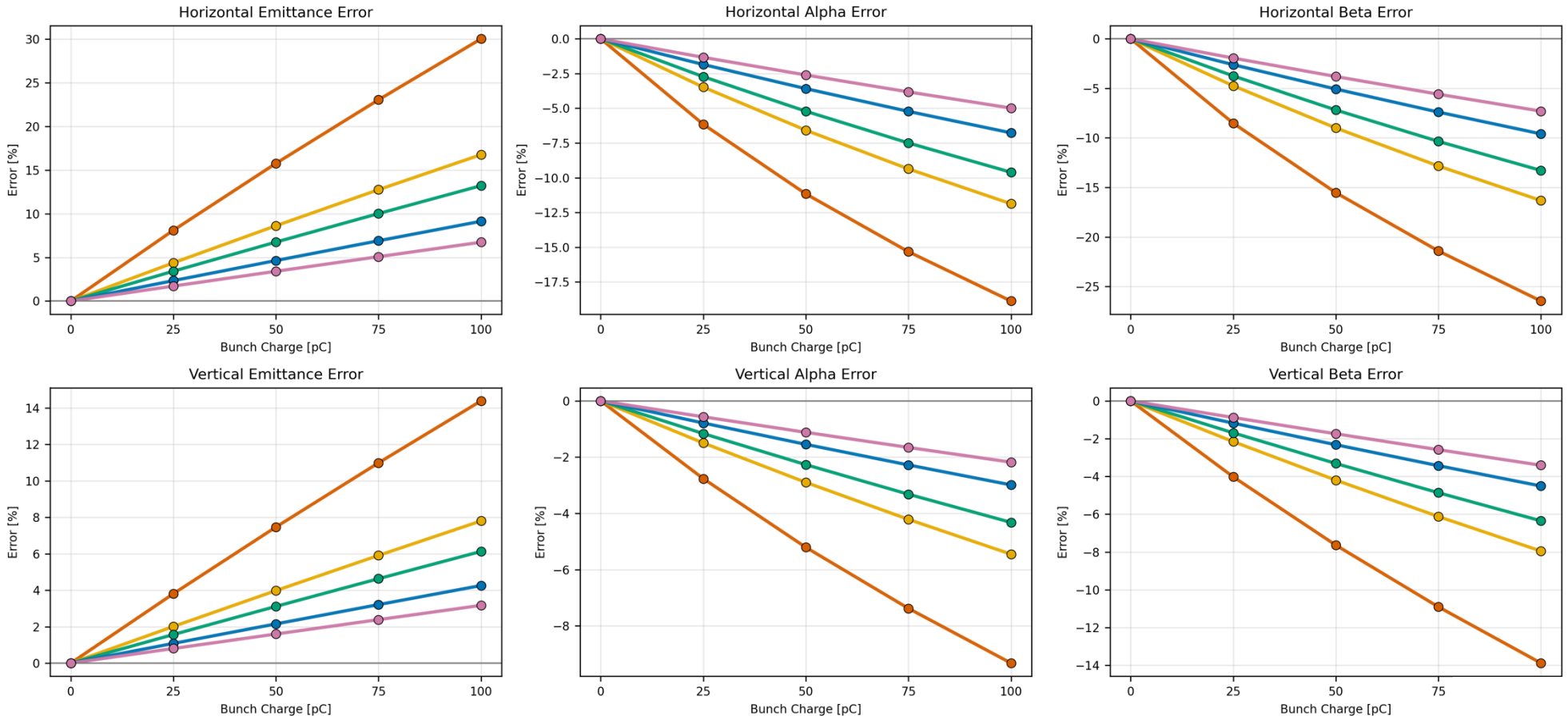




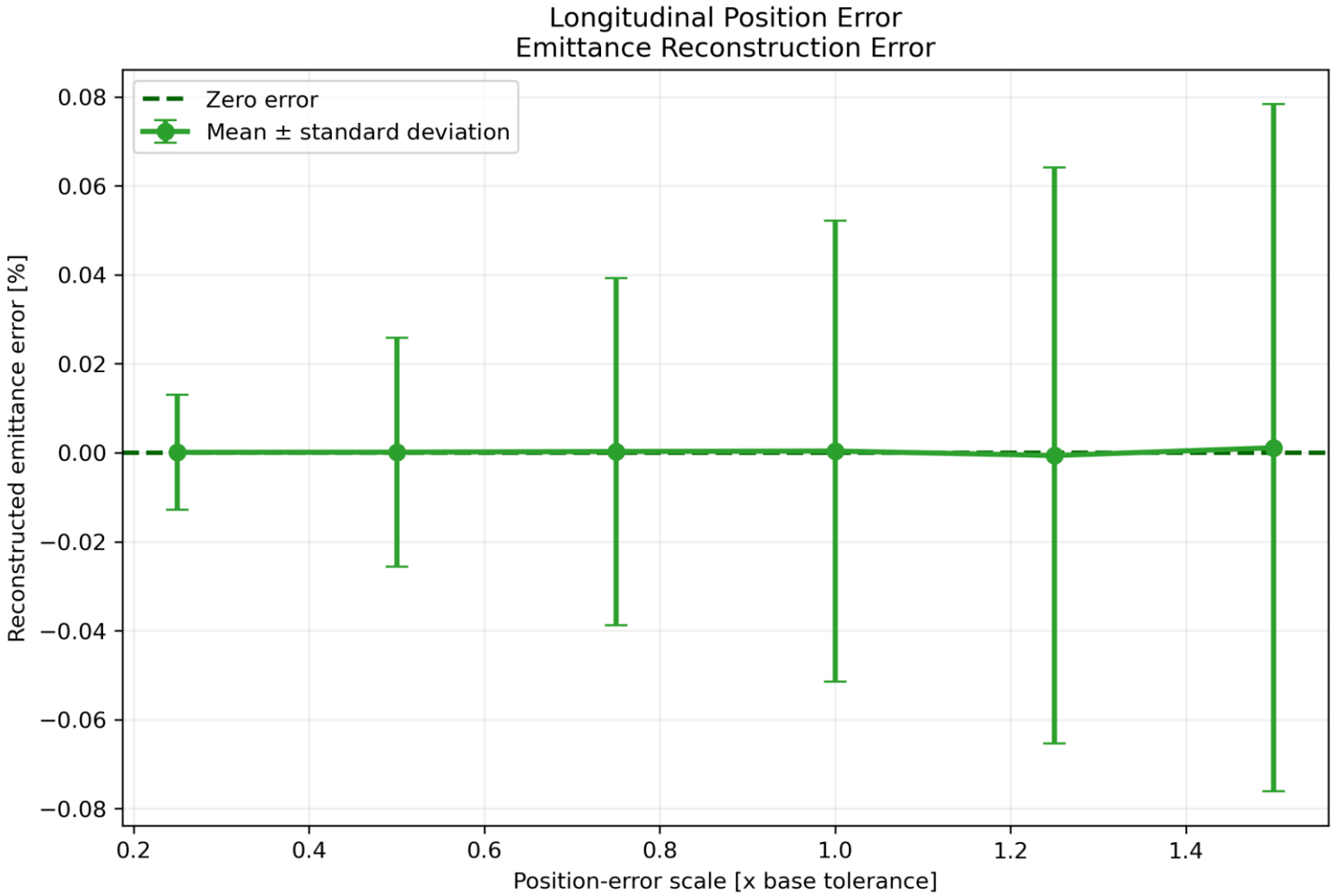
RESULTS OF DIFFERENT BEAMS

Energy / Charge	X emit [mm mrad]	X emit error [%]	X alpha	X alpha error [%]	X beta [m]	X beta error [%]	Y emit [mm mrad]	Y emit error [%]	Y alpha	Y alpha error [%]	Y beta [m]	Y beta error [%]
8.37 MeV - Reference	0.582	---	-1.901	---	4.305	---	0.608	---	-2.100	---	4.798	---
8.37 MeV - 0 pC	0.582	0.000	-1.901	-0.000	4.305	-0.000	0.608	-0.000	-2.100	0.000	4.798	0.000
8.37 MeV - 25 pC	0.630	8.106	-1.784	-6.155	3.938	-8.520	0.631	3.809	-2.042	-2.766	4.605	-4.024
8.37 MeV - 50 pC	0.674	15.777	-1.689	-11.162	3.637	-15.528	0.653	7.468	-1.990	-5.208	4.431	-7.638
8.37 MeV - 75 pC	0.717	23.059	-1.610	-15.320	3.383	-21.410	0.675	10.990	-1.945	-7.378	4.275	-10.905
8.37 MeV - 100 pC	**0.757**	**30.047**	**-1.543**	**-18.868**	**3.166**	**-26.462**	**0.695**	**14.391**	**-1.904**	**-9.323**	**4.132**	**-13.883**
11.79 MeV - Reference	0.421	---	-1.972	---	4.371	---	0.439	---	-2.178	---	4.871	---
11.79 MeV - 0 pC	0.421	-0.000	-1.972	0.000	4.371	0.000	0.439	-0.000	-2.178	0.000	4.871	0.000
11.79 MeV - 25 pC	0.439	4.380	-1.903	-3.480	4.163	-4.746	0.448	2.011	-2.146	-1.497	4.765	-2.158
11.79 MeV - 50 pC	0.457	8.630	-1.842	-6.579	3.977	-8.999	0.456	3.980	-2.115	-2.898	4.666	-4.196
11.79 MeV - 75 pC	0.474	12.760	-1.787	-9.361	3.810	-12.837	0.465	5.910	-2.087	-4.213	4.572	-6.124
11.79 MeV - 100 pC	0.491	16.775	-1.738	-11.871	3.657	-16.319	0.473	7.804	-2.060	-5.450	4.483	-7.952
13.37 MeV - Reference	0.373	---	-1.949	---	4.326	---	0.389	---	-2.151	---	4.819	---
13.37 MeV - 0 pC	0.373	0.000	-1.949	-0.000	4.326	-0.000	0.389	0.000	-2.151	-0.000	4.819	-0.000
13.37 MeV - 25 pC	0.386	3.420	-1.895	-2.726	4.163	-3.748	0.395	1.569	-2.126	-1.164	4.738	-1.694
13.37 MeV - 50 pC	0.398	6.757	-1.847	-5.213	4.015	-7.185	0.401	3.113	-2.102	-2.269	4.660	-3.314
13.37 MeV - 75 pC	0.410	10.023	-1.802	-7.496	3.878	-10.353	0.407	4.633	-2.080	-3.319	4.585	-4.864
13.37 MeV - 100 pC	0.422	13.223	-1.761	-9.604	3.751	-13.286	0.413	6.131	-2.058	-4.320	4.513	-6.351
16.03 MeV - Reference	0.313	---	-1.872	---	4.194	---	0.327	---	-2.063	---	4.670	---
16.03 MeV - 0 pC	0.313	0.000	-1.872	-0.000	4.194	-0.000	0.327	0.000	-2.063	-0.000	4.670	-0.000
16.03 MeV - 25 pC	0.320	2.340	-1.837	-1.851	4.084	-2.612	0.330	1.082	-2.046	-0.787	4.615	-1.179
16.03 MeV - 50 pC	0.327	4.641	-1.805	-3.587	3.981	-5.072	0.334	2.152	-2.031	-1.546	4.562	-2.322
16.03 MeV - 75 pC	0.334	6.909	-1.774	-5.224	3.883	-7.396	0.337	3.211	-2.016	-2.279	4.510	-3.431
16.03 MeV - 100 pC	0.341	9.142	-1.745	-6.767	3.791	-9.594	0.340	4.259	-2.001	-2.987	4.459	-4.508
18.37 MeV - Reference	0.274	---	-1.785	---	4.043	---	0.286	---	-1.962	---	4.499	---
18.37 MeV - 0 pC	0.274	-0.000	-1.785	0.000	4.043	0.000	0.286	-0.000	-1.962	0.000	4.499	0.000
18.37 MeV - 25 pC	0.279	1.715	-1.761	-1.333	3.964	-1.946	0.288	0.802	-1.950	-0.567	4.460	-0.882
18.37 MeV - 50 pC	0.283	3.410	-1.738	-2.605	3.889	-3.807	0.291	1.597	-1.940	-1.119	4.421	-1.743
18.37 MeV - 75 pC	0.288	5.085	-1.717	-3.821	3.817	-5.591	0.293	2.387	-1.929	-1.656	4.383	-2.585
18.37 MeV - 100 pC	0.293	6.742	-1.696	-4.985	3.748	-7.302	0.295	3.170	-1.919	-2.180	4.346	-3.409

ERROR STUDY OF DIFFERENT BEAMS



Positioning Error Study

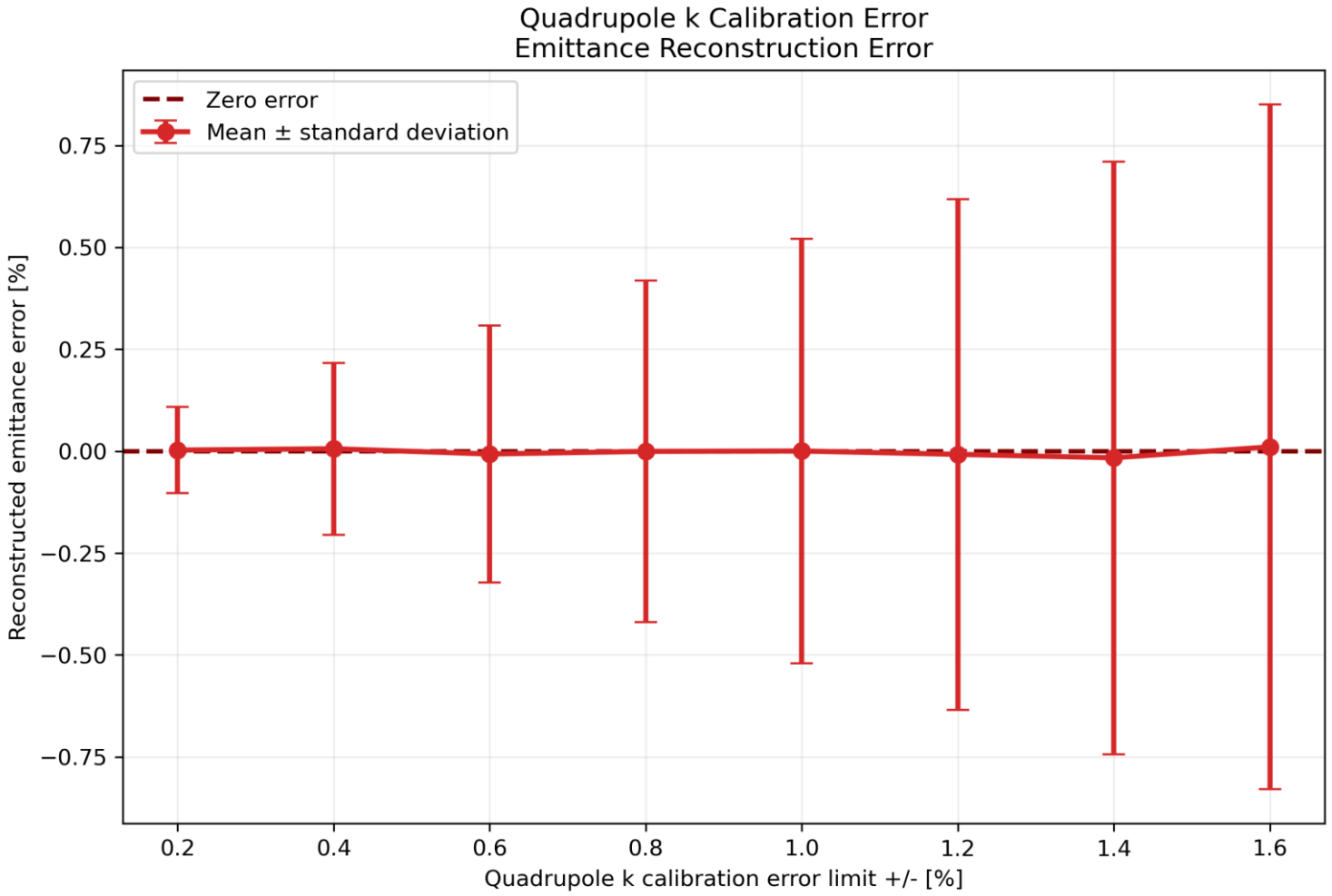


Uniform distribution

Q1/Q2 position error: ±5 to ±30 μm

Screen position error: ±0.125 to ±0.75 mm

Quadrupole Strength Error Study

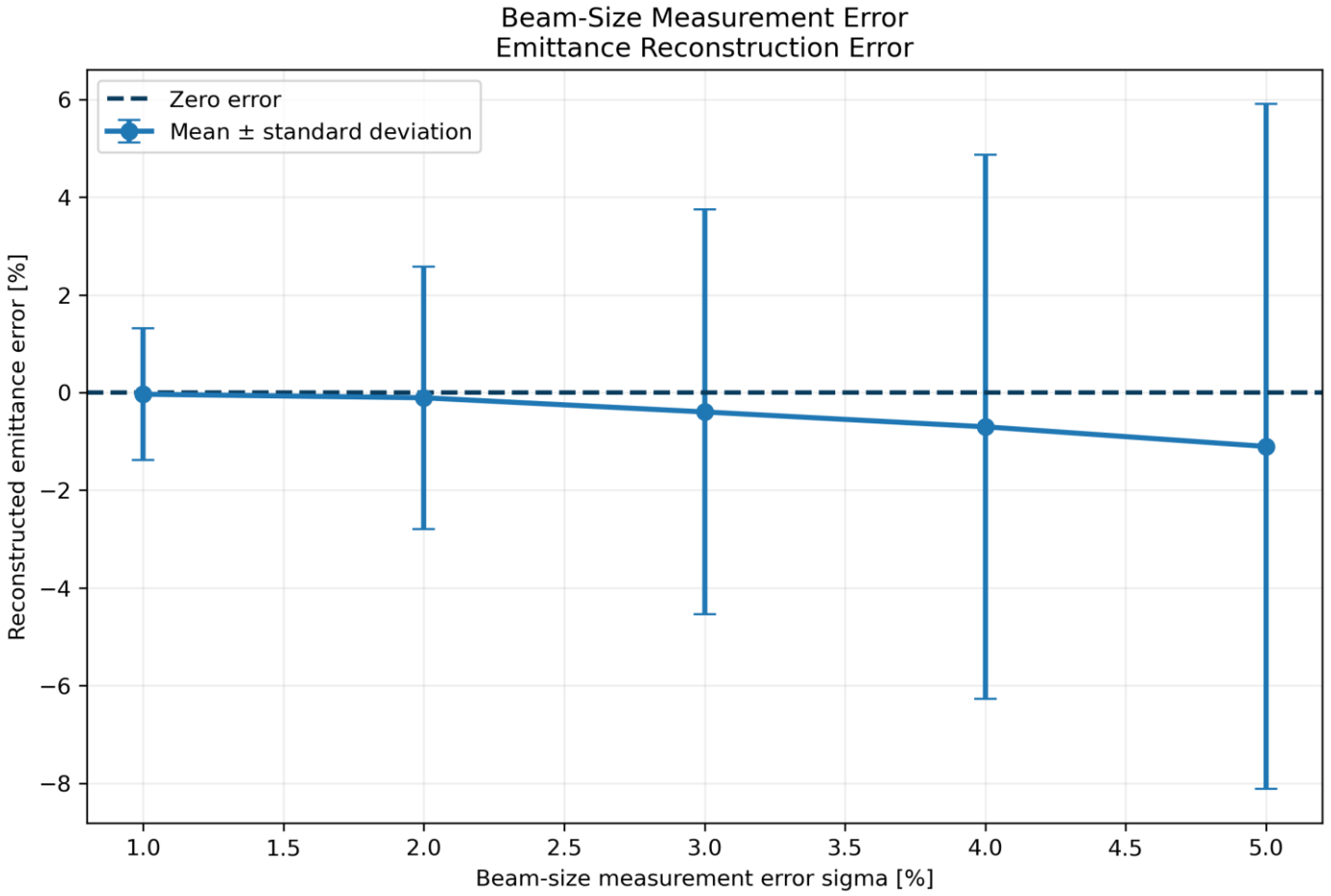


Uniform distribution

Independent errors for Q1 and Q2

Error range: ±0.2% to ±1.6%

Beam Size Error Study

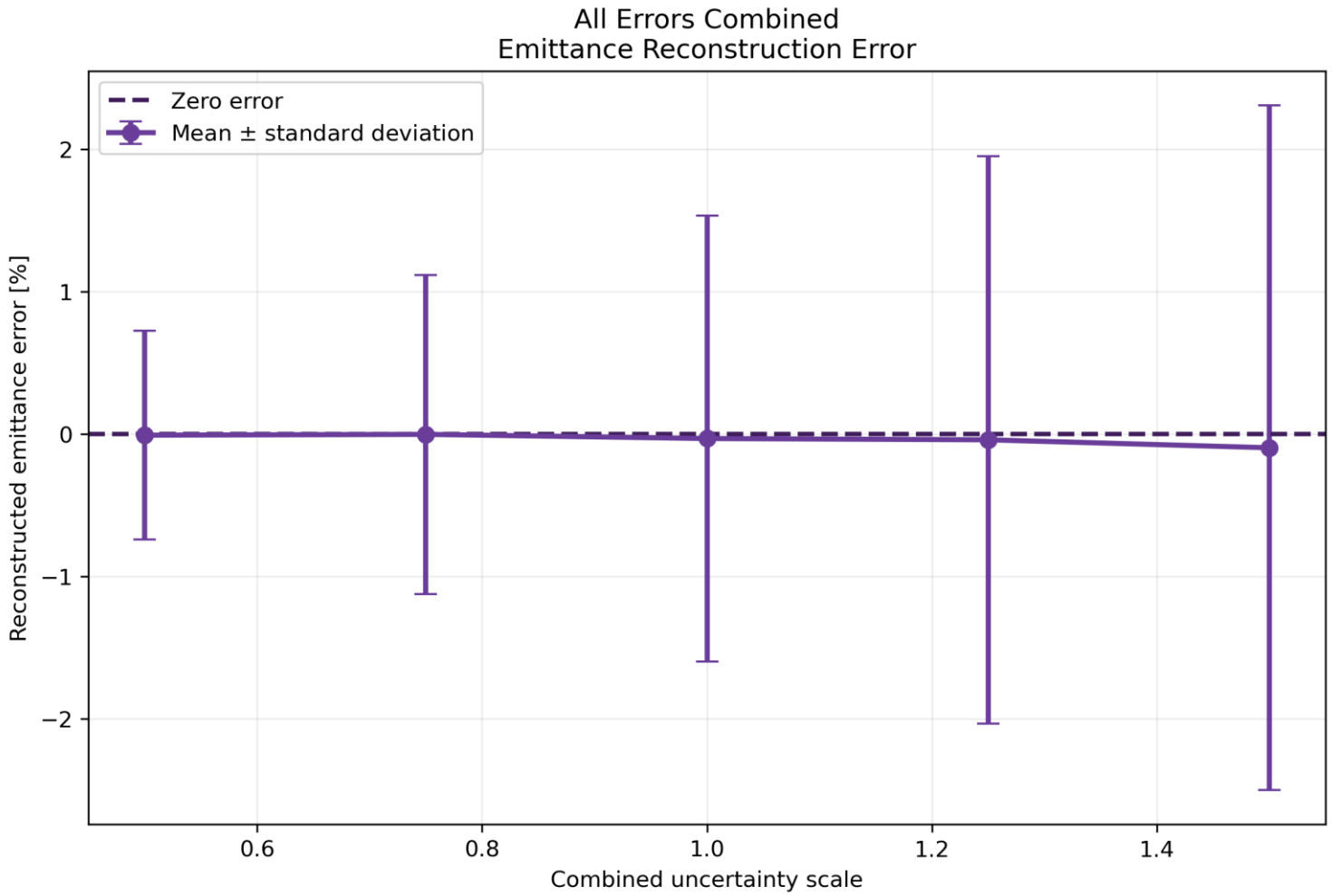


Gaussian distribution

Applied to measured beam size,

σ Error level: 1% to 5%

Combined Error Study



All error sources applied simultaneously

Beam size: 0.5–1.5%

GaussianQuadrupole k: ±0.5–±1.5%

UniformQ1/Q2 position: ±10–±30 μm

Screen position: ±0.25–±0.75 mm